Continental Shelf Research 48 (2012) 157–166

Contents lists available at SciVerse ScienceDirect

Continental Shelf Research

journal homepage: www.elsevier.com/locate/csr

Research papers

Prediction of cross-shore beach profile evolution using a diffusion type model

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ARTICLE INFO

Article history:
Received 23 August 2011
Received in revised form
17 July 2012
Accepted 3 August 2012
Available online 10 August 2012

Keywords:
Coastal morphology
Beach profile evolution
Behaviour-oriented model
Diffusion equation
Canonical correlation analysis
Christchurch Bay

ABSTRACT

A medium term morphodynamic prediction model for beach profiles, based on a 1-D diffusion formulation, is demonstrated here. The model combines an inverse methodology with a data-driven approach to derive unknown key parameters in the model governing equation. The field site used to demonstrate the model is Milford-on-Sea beach located within the Christchurch Bay beach system on the south coast of the UK, where historic measurements of cross-shore beach profile surveys and incident waves have been recorded over two decades. Despite the simplicity of the modelling approach, the model gives encouraging predictions of cross-shore beach profile changes at Milford-on-Sea. The predictive ability of the model is tested by forecasting measured beach changes on the basis of parameter calibrations performed on an independent set of measurements. Comparisons are also made against the results of a purely data-driven technique. In both cases the new method shows measureable improvements.

1. Introduction

Coastal flooding, erosion and stability of beaches and sea defence structures are extremely sensitive to the response of beach profiles to external morphodynamic drivers. Beach profile change is primarily linked to cross-shore sediment transport but, longshore transport can also play a role in determining long term profile change. Historically, cross-shore and longshore processes were modelled separately, relying on the engineering judgement to assess three dimensional effects. Even though recent developments led to 3D process-based morphodynamic models, they are still at early stages of development and also, extremely computationally intense. Therefore, simplified models of medium to long term changes of beach fronts still provide coastal engineers and managers, helpful guidance at a reasonable effort in terms of computational costs and data requirements for boundary conditions.

Predicting cross-shore beach profile evolution is primarily based on four approaches. The first approach is the use of equilibrium concepts where beach profiles are assumed to be in a dynamic equilibrium state as a result of balance between constructive and destructive forces applied on them. Bruun (1954) developed the earliest relationship between profile depth and offshore distance of the equilibrium profile shape, as an empirical formula. Dean (1977) then provided a physical argument for the profile shape taking wave energy dissipation into account and related the profile shape to the beach sediment characteristics. Later, Dean (1991) included gravity effects to get the linear upper beach and also retain the dependence on grain size. Bodge (1992) and Inman et al. (1993) proposed alternative formulations for the equilibrium profile to eliminate non-physicality of the profile at the shoreline where beach slope is infinite. Equilibrium profile formulations are commonly used to determine long-term beach profile forms.

The second approach is the use of process-based models based on hydrodynamic equations combined with sediment transport and morphodynamic modules (e.g., Reniers et al., 1995, Roelvink et al., 2009; Southgate and Nairn, 1993). These models are an extremely valuable tool for assessing local, short-term morphodynamic changes in a beach profile, but have inherent limitations due to our lack of knowledge of sediment transport processes and their linkage to hydrodynamics, uncertainties in hydrodynamic forcing and potential over-sensitivity to initial and boundary conditions, when applied to longer term predictions. The inaccuracies of the predicted profiles on longer term time scales are largely unknown.

The third group of models have been termed behaviour-oriented models. These aim to reproduce the behaviour of beach morphology using governing equations that are simplified to retain only key processes (e.g., Cowell et al., 1992, 1994; Stive and de Vriend, 1995; Reeve and Fleming, 1997; Hanson et al., 2003). The governing equations used in these models are rarely

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http://dx.doi.org/10.1016/j.csr.2012.08.004

derived from first principles; rather, they are defined along the lines of physical arguments. These models are effectively used in determining beach evolution over mid to long term time scales (i.e., years to decades).

Finally, the last group of methods is based on statistical analysis of historical measurements, with forecasts made on the basis of extrapolation (Różyński, 2003; Reeve et al., 2008). This approach has been termed 'data-driven' as it relies on statistical analysis of measurements with no solution of dynamical equations.

Morphology of beach profiles evolves at a range of time scales. The time scales of relevance are storm time scale of hours to days, seasonal time scale of months to an year and climate variability time scale of several years to few decades. A suitable model to determine beach profiles has to be selected based on the relevant time scale of importance and the level of sophistication required for a given situation.

Diffusion type formulations are widely used in behaviour-oriented models of beach plan shape changes (e.g., Pelnard-Considere, 1956; Reeve and Fleming, 1997; Hanson et al., 2003; Avdeev et al., 2004, 2009). In the diffusion formulations applied to cross shore profile changes, the profile depth is described as a function of cross-shore position, with appropriate initial and boundary conditions. This type of formulation is used to reproduce beach profile morphology on the basis that the solutions map the behaviour of the beach profile in a qualitative manner. Diffusion has the effect of smoothing irregularities in the profile. However, smoothing is not the only morphological response of a beach profile. Therefore, other morphological changes to the beach profile such as steepening of the profile and evolution of near-shore bars are included here as a source function in the equation which is an aggregation of changes driven by physical processes other than sediment diffusion.

In recent years, a novel hybrid approach was developed for beach morphology evolution modelling using an inverse technique. Karunarathna et al. (2008) developed a hybrid 2-D diffusion model to predict morphological evolution of estuaries, taking diffusion coefficient as a constant and demonstrated its application to the Humber Estuary (Reeve and Karunarathna, 2011). This model was capable of determining estuarine morphodynamic trends at time scale of several years to a decade. A 1-D hybrid diffusion model to predict cross-shore beach profile evolution can be found in Karunarathna et al. (2009). In Karunarathna et al. (2011), the key parameters of the above model were analysed and related to site conditions and morhodynamic drivers thus explaining the physical significance of the model.

The aim of this paper is to demonstrate a hybrid predictive beach profile model based on a combination of a behaviour-oriented and a data-driven statistical analysis of historic measurements. Model formulation is briefly discussed in Section 2. Section 3 of the paper presents a brief description of the field site used for model demonstration. Model results are presented and discussed, and compared with a direct data-driven model, in Section 4. Section 5 concludes the paper.

2. Cross-shore beach profile evolution model

Following Stive et al. (1991) and Hanson et al. (2003), the diffusion formulation used for cross-shore beach profile evolution in this research is given by

$$\frac{\partial h(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(K(x,t)\frac{\partial h(x,t)}{\partial x}\right) + S(x,t) \tag{1}$$

Eq. (1) describes time and space variation of profile depth $h(x,t)$ at a cross shore location x, where x is measured offshore from the mean water shoreline (MWL). Schematic of the model is given in Fig. 1. $K(x,t)$ in Eq. (1) is an unknown space- and time-dependent sediment diffusion coefficient. $S(x,t)$ is a source function to describe non-diffusive contribution to morphodynamic variability including all processes relating to changes in natural environmental forcing and human induced impacts. It is assumed here that cross-shore beach morphology evolves as a result of diffusive and non-diffusive sediment dynamic processes and that the contribution from longshore transport dynamics can be represented as a component within the source term.

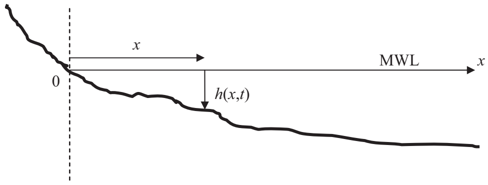

Fig. 1. Schematic of the cross-shore profile evolution model.

The application of Eq. (1) to predict beach profiles requires determination of the unknown parameters K and S. An inverse solution to Eq. (1) can be sought to determine both unknowns. Inverse solutions to diffusion type formulations are well founded in the literature. Cannon and Ewing (1976) presented a procedure for computing a source term in a linear diffusion equation. Cannon and Du Chateau (1980) gave an inverse solution to determine the diffusion coefficient in a non-linear diffusion equation. Reeve and Spivack (1994) and Spivack and Reeve (2000) presented an inverse method based on a split step method to determine a source function in a linear diffusion equation in one and then two space dimensions. This approach also formed the basis of a method for finding a spatially varying coefficient in a diffusion equation without any source term, (Spivack and Reeve, 1999). Avdeev et al. (2009) describe a method for simultaneously deriving a source and coefficient, based on optimisation techniques, in a coastal profile diffusion equation. The equation they investigate has a slightly different form to Eq. (1), the most important difference being that the varying diffusion coefficient is not included within the derivative.

Here we follow the approximate method described in Karunarathna et al. (2009) to determine the two unknowns. First, both K and S are taken as a sum of time varying and time averaged components as in Reynolds expansion:

$$h(x,t) = \overline{h}(x) + h'(x,t)$$

$$K(x,t) = \overline{K}(x) + K'(x,t)$$

$$S(x,t) = \overline{S}(x) + S'(x,t)$$

here, over-bar denotes the time averaged components and prime denotes the time varying residuals.

Then, Eq. (1) can be re-written as

$$\frac{\partial\left[\overline{h}(x)+h'(x,t)\right]}{\partial t} = \frac{\partial}{\partial x}\left(\left[\overline{K}(x)+K'(x,t)\right]\frac{\partial\left[\overline{h}(x)+h'(x,t)\right]}{\partial x}\right) + \overline{S}(x) + S'(x,t) \tag{2}$$

If we include time-varying component of K in S, Eq. (2) can then be written as

$$\frac{\partial\left[\overline{h}(x)+h'(x,t)\right]}{\partial t} = \frac{\partial}{\partial x}\left(\overline{K}(x)\frac{\partial h(x,t)}{\partial x}\right) + \overline{G}(x) + G'(x,t) \tag{3}$$

where

$$\overline{G}(x) + G'(x,t) = G(x,t) = \frac{\partial}{\partial x}\left(K'(x,t)\frac{\partial\left[h(x,t)\right]}{\partial x}\right) + S(x,t) \tag{4}$$

Then, Eq. (3) can be rewritten in the form:

$$\frac{\partial h(x,t)}{\partial t}=\frac{\partial}{\partial x}\left(\overline{K}(x)\frac{\partial h(x,t)}{\partial x}\right)+G(x,t) \tag{5}$$

The variable $G(x,t)$ in Eq. (5) contains $S(x,t)$ and time-varying diffusive effects. $\overline{K}$ and $G(x,t)$ are the key parameters that determine the ability of the model in predicting cross-shore beach profile evolution in time. If $\overline{K}$ and $G(x,t)$ can be resolved then, Eq. (5) can be used as a predictive model for cross-shore beach profile change.

$\overline{K}$ and $G(x,t)$ are site specific and directly linked with the sediment properties and exposure of a given site (Karunarathna et al., 2011). The calculation of $\overline{K}$ and $G(x,t)$ using Eq. (5) and beach profiles at two different times represents a challenging, generally ill-posed inverse problem. An approximate, two stage, process is used here. The mathematical framework used to determine $\overline{K}$ and $G(x,t)$, when sufficient historic data of cross-shore beach morphology at a given site is available, is described in detail in Karunarathna et al. (2009). A description of the methodology is summarised here for clarity.

First, we take time average of Eq. (5), assuming that the time period taken for time averaging is sufficiently long. Then, the time averaged Eq. (5) takes the form

$$0=\frac{\partial}{\partial x}\left(\overline{K}(x)\frac{\partial \overline{h}(x)}{\partial x}\right)+\overline{G}(x) \tag{6}$$

where $\overline{h}$ and $\overline{G}(x)$ are the time-mean components of $h(x,t)$ and $G(x,t)$, respectively. An analogy to the Reynolds' stresses of turbulent fluid flow, $\overline{G}(x)$ may be considered to be a turbulent morphodynamic stress. As a first order approximation, these stresses are taken to be zero. If Eq. (6) is solved for the time-averaged cross-shore beach profile, a solution for time-mean diffusion coefficient can be derived as

$$\overline{K}(x)=\frac{\alpha}{\left((\partial\overline{h}(x)/\partial x)\right)} \tag{7}$$

where α is a constant of integration, (for the denominator $\neq 0$). The time averaged diffusion coefficient $\overline{K}(x)$ may then be substituted into Eq. (5) to derive an estimate for the source function $G(x,t)$ using a first order approximation.

The approximate solution of Eq. (5) to determine G, assuming the diffusion coefficient is known, (see Spivack and Reeve, 2000), gives:

$$G(x,t)=\tfrac{1}{\tau}\left[h(t+\tau)-\exp(\tau D)h(t)\right] \tag{8}$$

In Eq. (8), $h(t)$ and $h(t+\tau)$ are two consecutive bathymetries in a time series of cross-shore profile surveys measured at time interval τ apart. D is the operator

$$D=\frac{\partial}{\partial x}\left(\overline{K}\frac{\partial}{\partial x}\right) \tag{9}$$

A time series of $G(x,t)$ corresponding to each consecutive pair of cross-shore profile surveys can be derived using Eq. (8).

A detailed analysis of $\overline{K}$ and G was carried out in Karunarathna et al. (2011) in order to assess their contribution to cross-shore morphology change. A comparison of diffusion coefficient with Dean's equilibrium beach profile showed that it is directly linked to sediment properties of and the physical nature of the beach concerned. The source function demonstrated a strong correlation to the incident waves measured at the site.

If the diffusion coefficient and the source function are specified, Eq. (5) may be solved to march a solution forward in time to determine future beach profiles. The quality of forecasts made in this way will clearly depend upon how well the source function can be specified. Such an approach might succeed should (i) the source function vary slowly over time, (ii) the range of variation contained in the historic records bounds that in the forecast period. To determine the solution, we write Eq. (5) in operator notation:

$$h_t=Dh+G \tag{10}$$

where h_t is the time derivative of h.

If the time variation of $G(x,t)$ over one model time step is weak then, the formal solution of Eq. (10) can be obtained as (Spivack and Reeve, 2000)

$$h(x_i,t_{j+1})\cong(\exp(D\tau)-1)D^{-1}G+\exp(D\tau)h(x_i,t_j) \tag{11}$$

which gives the model governing equation of cross-shore beach variability in predictive form, where $h(x_i,t_j)$ and $h(x_i,t_{j+1})$ are the profile depth at the ith cross-shore node at jth an $(j+1)$th time steps, respectively.

3. Field site and historic data

To demonstrate the application of the diffusion model, the Milford-on-Sea beach at Christchurch Bay, UK, which has a comprehensive set of historic cross-shore beach profile surveys and incident waves, is selected in this study. Christchurch Bay is regarded as a self-contained sediment system with limited sediment input from offshore sources cliff erosion (Halcrow Group, 1999). Milford-on-Sea, located at the eastern end of Christchurch Bay, is a composite sand-gravel beach with complex and highly variable cross-shore morphology. The sediment grain size at Milford-on-Sea beach varies significantly along the cross shore profile. Coarse shingle and pebbles with a median grain diameter (D_{50}) around 14 mm dominate the upper beach. A sand- gravel mix which has D_{50}-gravel = 10 mm and D_{50}-sand = 1 mm with only 12% sand fraction, dominates inter-tidal areas (Martìn-Grandes et al., 2009). The location has a modest tidal range of 2.0 m at spring tide. It is primarily wave-dominated. A map of the beach and its location in the UK and, the location of profile and wave measurements used in this study are given in Fig. 2.

As a part of the UK national programme of shoreline management planning, cross-shore beach profiles at Christchurch Bay have been monitored by the New Forest District Council at a number of locations since 1986. All profile surveys have been carried out with reference to the Ordnance Survey Datum, Newlyn (ODN).

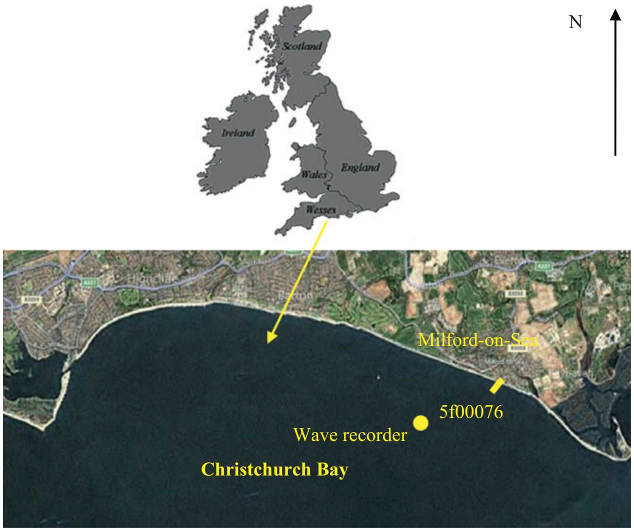

Fig. 2. Milford-on-Sea, it is location in the UK, the location of beach profile surveys used in the model and location of nearshore wave measurement.

From a detailed sediment transport study carried out by the Standing Conference on Problems Associated with the Coastline (SCOPAC), 2003, the lowest external sediment input–output and longshore transport were observed around the cross-shore profile location 5f00076 at Milford-on-Sea (Fig. 2). This indicates that profile change at location 5f00076 is dominated by cross-shore sediment exchange. Therefore, cross-shore profile surveys measured at the transect 5f00076 are used here to demonstrate the model. Beach profile surveys from 1986 to 2005 and wave measurements for the same period were obtained from the Channel Coastal Observatory (CCO). Fig. 3 shows cross-shore profile surveys used in this study. The measurements between dune top and mean low water level (MLWS) measured with respect to ODN were used in the analysis.

Wave data at Christchurch Bay, measured in 10–12 m water depth, are available from 1986. The predominant wave direction is SSW. A typical wave height and period time series is given in Fig. 4. The most frequently occurring wave heights and periods at Milford-on-Sea are between 0.1–1.0 m and 4–6 s, respectively.

Before using cross-shore beach profile survey data for recovering unknown model parameters as described in Section 2, all data are rearranged to make them amenable to numerical analysis. The data preparation and rearrangement procedures are described in detail in Karunarathna et al. (2011). Processed data contains beach profiles from the dune crest up to the MLWS at uniformly spaced cross-shore intervals and time of 0.5 m and 90 day, respectively.

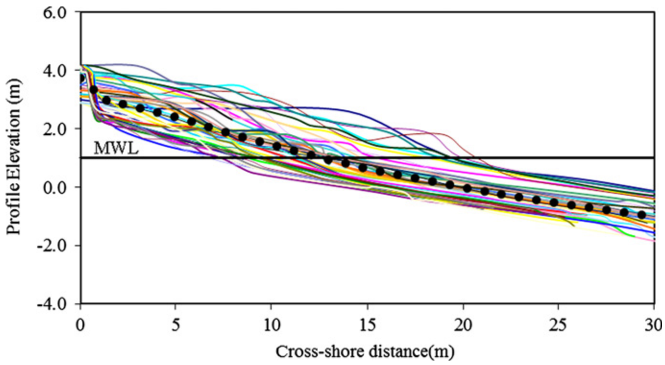

Fig. 3. Cross-shore profiles measured at transect 5f00076 at Milford-on-Sea beach from 1987 to 2005. Mean profile is shown in black dotted time.

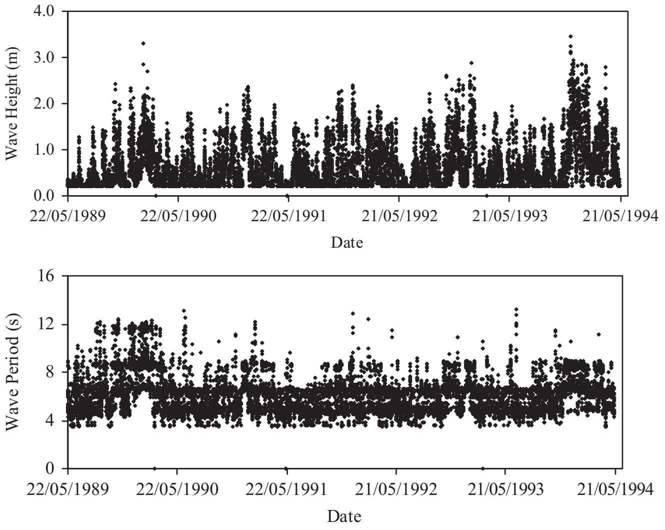

Fig. 4. A typical wave height and wave period time series measured at Christchurch Bay.

4. Model demonstration and discussion

The two unknown model parameters, diffusion coefficient K and the source function G in Eq. (5) were determined by following the mathematical procedure described in Section 2, using the measured cross-shore beach profiles at transect 5f00076 for the period 1987–1994, described in Section 3. Following that, the source functions were correlated to incident waves concurrently measured at the site over the same period. The results were then used in Eq. (11) to determine beach profiles in the period 1999–2005.

4.1. Sediment diffusion coefficient

The time-averaged component of the sediment diffusion coefficient at beach transect 5f00076 can be determined from Eq. (4) given in Section 2. First, the cross-shore gradient of the time-mean beach profile $\partial\overline{h}/\partial x$ was obtained using pre-processed historic beach profile surveys at 5f00076. In order to determine $\overline{K}$ from Eq. (4), the integration constant α should be known. An optimum value for α is determined by (i) choosing a range of α values covering few orders of magnitude (ii) calculating corresponding $\overline{K}$ using Eq. (7), (iii) solving Eq. (5) for $h(x,t)$ by taking $G(x,t)=0$ as an initial approximation, (iv) calculating the root mean square error between computed and measured $h(x,t)$ and (v) selecting the α which gives minimum root mean square error between measured and computed $h(x,t)$. Note that to avoid the occurrence of indeterminate $\overline{K}$ values, where the average cross-shore profile gradient is zero, either the average value of the mean gradient at immediately neighbouring points or a weighted average of the mean gradients at, say, four or six neighbouring points can be used to determine $\overline{K}$. A detailed description of the computational procedure is given in Karunarathna et al. (2009).

Fig. 5 gives the cross-shore variability of time mean diffusion coefficient against mean profile depth at beach transect 5f00076. The general trend here is that $\overline{K}$ gradually increases with profile depth. This reflects the variability of beach sediment size across the profile where coarse gravel with lower sediment diffusion coefficient dominates the upper beach and a mix of sand and gravel with a higher diffusion coefficient exist in the inter-tidal beach (Martìn-Grandes et al., 2009). $\overline{K}$ varies between 10^{-3} and 10^{-2} m^2/day, which is of the same order of magnitude found by previous researchers at similar sites (Masselink and Pattiarachchi, 1998).

4.2. Source function

The time mean diffusion coefficient given in Fig. 5 is then used to determine the time- and space-varying source function for each consecutive pair of bathymetry surveys, using Eq. (8). The maximum, minimum and average source function across the profile 5f00076 are shown in Fig. 6. According to Fig. 6, the source

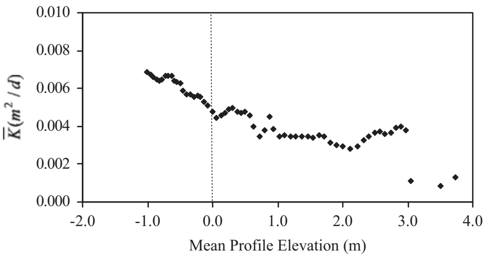

Fig. 5. Variability of mean diffusion coefficient with time mean profile elevation at transect 5f00076.

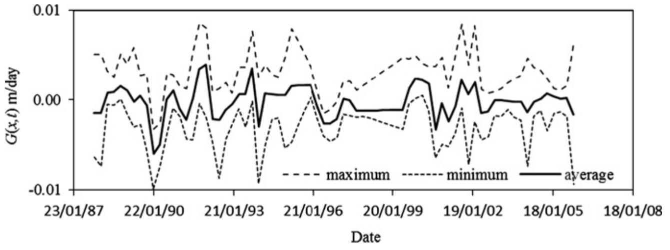

Fig. 6. Time history of the envelope of the source functions across the cross-shore profile at 5f00076.

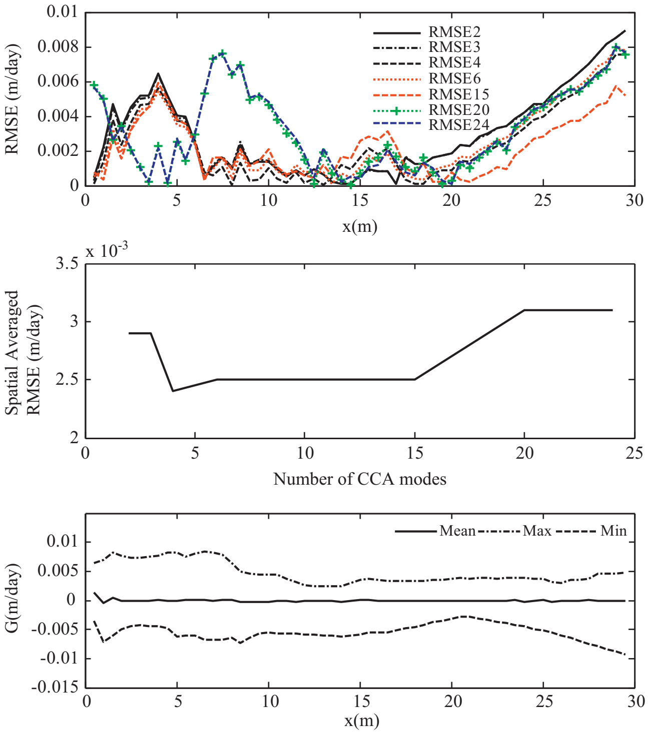

Fig. 7. Cross-shore variability of time averaged RMSE between CCA-forecasted and data-derived source functions (top panel); variation of cross-profile averaged RMSE as a function of the number of CCA modes (middle panel); and cross-shore variability of source function envelope (bottom panel) for transect 5f00076.

functions appear to capture alternate erosion (negative source function) and accretion (positive source function) of the cross-shore profile at seasonal/yearly timescales.

The challenge of using Eq. (11) as a predictive model for beach profile variability involves the selection of a suitable parameterisation of the time and space varying source function from the source functions recovered from historic data. One possible approach is to use historical data in a Monte-Carlo ensemble forecasting scheme (e.g., Reeve et al., 2008). Another approach would be to determine a direct correlation between the source functions and the external morphodynamic process drivers and then use that correlation to determine future source functions on the basis of knowledge or estimates of the wave conditions, (e.g., Horrillo-Caraballo and Reeve, 2010).

Here we follow the second approach. Milford-on-Sea being a wave dominated beach, waves are the primary external morphodynamic process driver. In order to establish a correlation between the source function and the incident waves, canonical correlation analysis (CCA) was performed between the two variables, taking the first part (1987–1994) of the source function and wave data series. The remaining part (1999–2005) of the source functions were

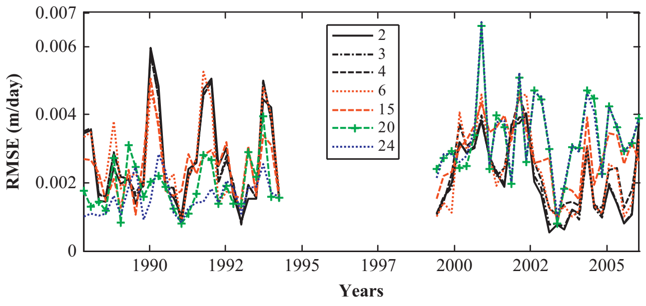

Fig. 8. Time series of cross-shore profile averaged RMSE between CCA derived and data derived source functions at transect 5f00076 for canonical modes 2,3,4,6,15,20 and 24.

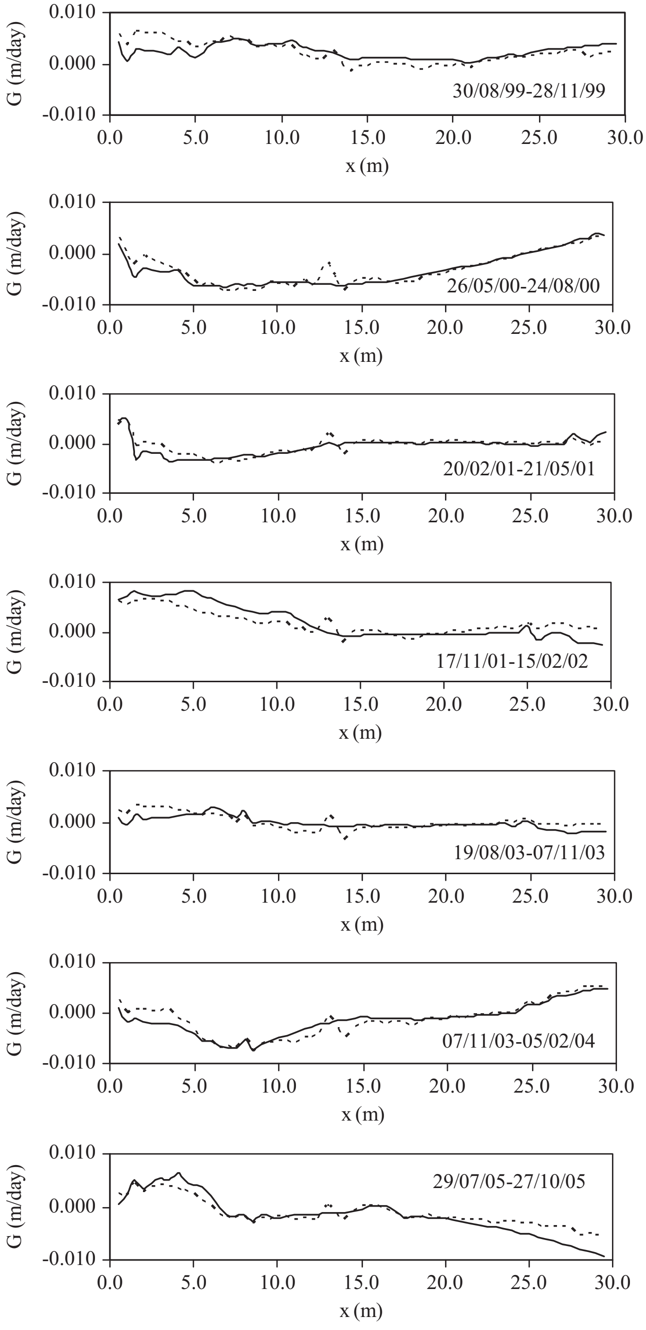

Fig. 9. Comparison of a selection of CCA-derived and data-derived source functions at transect 5f00076.

kept for performance verification of the methodology. The source functions between 1995 and 1998 were discarded as they may be contaminated by the beach refilling undertaken at Milford-on-Sea during that period (SCOPAC, 2003). CCA determines any patterns that tend to occur simultaneously in two different data sets and the correlation that exists between the associated patterns. A brief

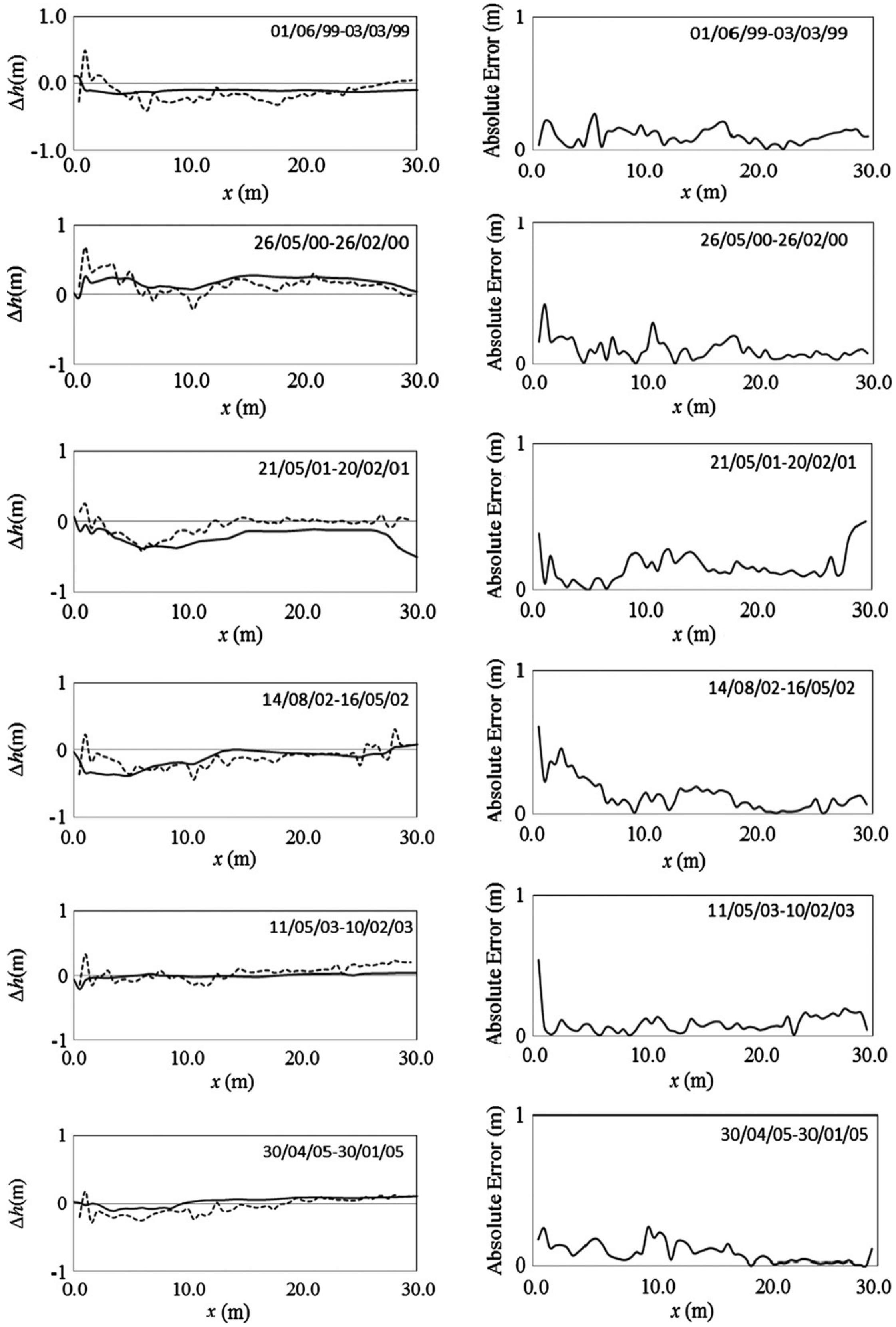

Fig. 10. Comparison of predicted cross-shore beach profile change by the diffusion model and the measured profiles and absolute errors.

outline of the technique is provided here but further details may be found in Clark (1975) and Różyński (2003).

As part of the CCA, a regression matrix (ψ) is determined which relates the source function to incident waves. If the matrix X contains wave data and the matrix Y contains source functions, then a linear combination of X and Y is sought to obtain the new variables U and V that are maximally correlated for the same index and zero correlation for differing indices. Then a matrix that contains predicted source terms (Y_p) for a period t_p can be obtained by multiplying the regression matrix ψ by a matrix X_p containing known or forecast wave conditions over the same period t_p.

To perform the CCA we require two time series with the same number of realizations. Source functions and waves at Milford-on-Sea are not available at the same frequency. In order to generate a time series of waves at the same frequency to that of the source functions, the wave measurements between successive beach profiles were combined to create probability density functions (pdf). Rather than fit a parametric form to the pdfs, empirical pdfs were used as suggested by Rihouey (2004). As noted in Karunarathna et al. (2011) a better correlation was found between the source functions and wave steepness than between source functions and wave height. In the following we have used wave steepness in the CCA.

When performing the predictions, a choice has to be made as to how many CCA modes are used. Truncating the summation over modes acts to filter out small scale variations. A threshold of 97% was used to define the number of retained CCA modes. Predictions with 2, 3, 4, 6, 15, 20 and 24 CCA modes were carried out in order to investigate the sensitivity of the results to the choice of truncation.

The performance of the CCA approach was investigated in detail by examining the root mean square error (RMSE) between the predicted and data-derived source functions for the period 1999–2005. The RMSE was calculated in two ways. First, as the error at a particular cross-shore location averaged over the forecasts in the period 1999–2006, and secondly as an average over the profile for a particular forecast. Cross-shore variability of the time-averaged RMSE between the forecasted and data-derived source functions (1999–2006) is shown in Fig. 7 (top panel). Overall, the largest RMSEs were seen in the supra-tidal beach face (x=1–5 m) and in the sub-tidal region (x=25–30 m), except for the cases with 20 and 24 canonical modes. The envelope of the source functions predicted from all cases considered and their overall mean are shown in the bottom panel of Fig. 7. As the RMSE considerably varies across the profile, the spatial (cross-shore) averages for all cases were then calculated (Fig. 7 middle panel) in order to determine the number of CCA modes that give the lowest overall RMSE across the profile. It can be seen that the RMSE reduces as the number of modes increases, plateaus and then increases as the number of modes exceeds 15. The lowest RMSE, averaged across the profile, is achieved by using 4 modes.

The time history of RMSE averaged across the cross-shore profile was also examined to investigate the accuracy of source function predictions in time. In Fig. 8, time series of cross-shore profile averaged RMSE is shown. The overall RMSE remains consistent and no specific trends are visible, irrespective of the number of canonical modes taken for predictions. However, it can be noted here also that fewer CCA modes consistently give lower RMSE in time. Based on the results shown in both Figs. 7 and 8, 4 canonical modes were selected for all source function predictions.

In Fig. 9, a selection of predicted source functions is compared with their computed counterpart. In all cases, the greatest discrepancies in the predictions occur in the swash region. Discrepancies can arise from the contribution of external morphodynamic drivers other than waves to the source function, which are not included in the analysis. Overall, the ability of the CCA in predicting the source function using wave steepness data is very satisfactory.

4.3. Profile prediction

In order to investigate the ability of the modelling approach adopted here, beach profile changes were predicted using source functions estimated from the wave measurements in the period 1999–2006 and the regression relation between waves and source functions found for the period 1987–1994, and the results were compared with measured beach profile change. The source functions determined from the CCA analysis and the mean diffusion coefficient determined directly from the historic beach profile surveys, (1987–1994), were used in time-stepping the model governing equation (Eq. (8)) to forecast beach profile changes from 1999 to 2005, taking the latest available measured profile as the initial profile.

Fig. 10 shows a selection of measured and predicted cross-shore profile change at 5f00076, with the value of absolute error between the two. Overall, the agreement between the predicted and measured profile changes is very encouraging, despite the simplicity of the modelling approach. In most cases, the largest discrepancy was observed in the swash zone where beach profile variability is higher than the rest of the profile on a steep beach.

In order to assess the overall performance of the model, the absolute value of the relative error of profile change predictions (the ratio of local error to maximum error along the profile at a given time step) is calculated for all predictions from 1999 to 2005 and the envelope of the relative error is determined. The results are shown in Fig. 11. Mean profile depth is also shown in the figure for clarity. It can be seen that highest prediction errors occurred at the upper beach face where swash oscillations take place. This could mainly be attributed to the processes that are not taken into account in determining the source functions using CCA. Those include ground water flow and infiltration-exfiltration that play a significant role in evolving complex upper beach morphology on a steep beach (Austin and Masselink, 2006; Jamal et al., 2010).

4.4. Comparison with a purely data-driven approach

It is natural to ask whether a reasonable prediction could not be achieved from a direct correlation between beach profiles and wave steepness, such as in Horrillo-Caraballo and Reeve (2010). To assess what benefit accrues from merging a data-driven approach with a diffusion type model, a corresponding set of predictions was generated through CCA between beach profiles and pdfs of wave steepness.

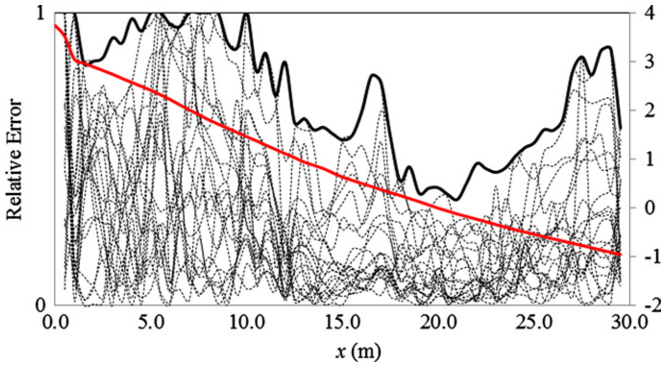

Fig. 11. The relative prediction errors of profile change predictions from the diffusion model for all cases from 1999 to 2005. Dark line shows the errors envelope. Mean profile is shown in solid red line for clarity.

A comparison of model predictions of beach profile change with measured data, for the same selection of cases shown in Fig. 10, is given in Fig. 12.Also, the mean absolute prediction error for all predicted cases from 1999 to 2005 is shown in Fig. 13. It is seen that the prediction errors of the most cases shown in Fig. 12 is higher than that of the diffusion model. Fig. 13 shows that even though both methods predict similar results in the swash zone, the diffusion model performed better in the intertidal zone.

The relative prediction errors of profile change predictions from the direct CCA approach for all cases (similar to Fig. 11 for

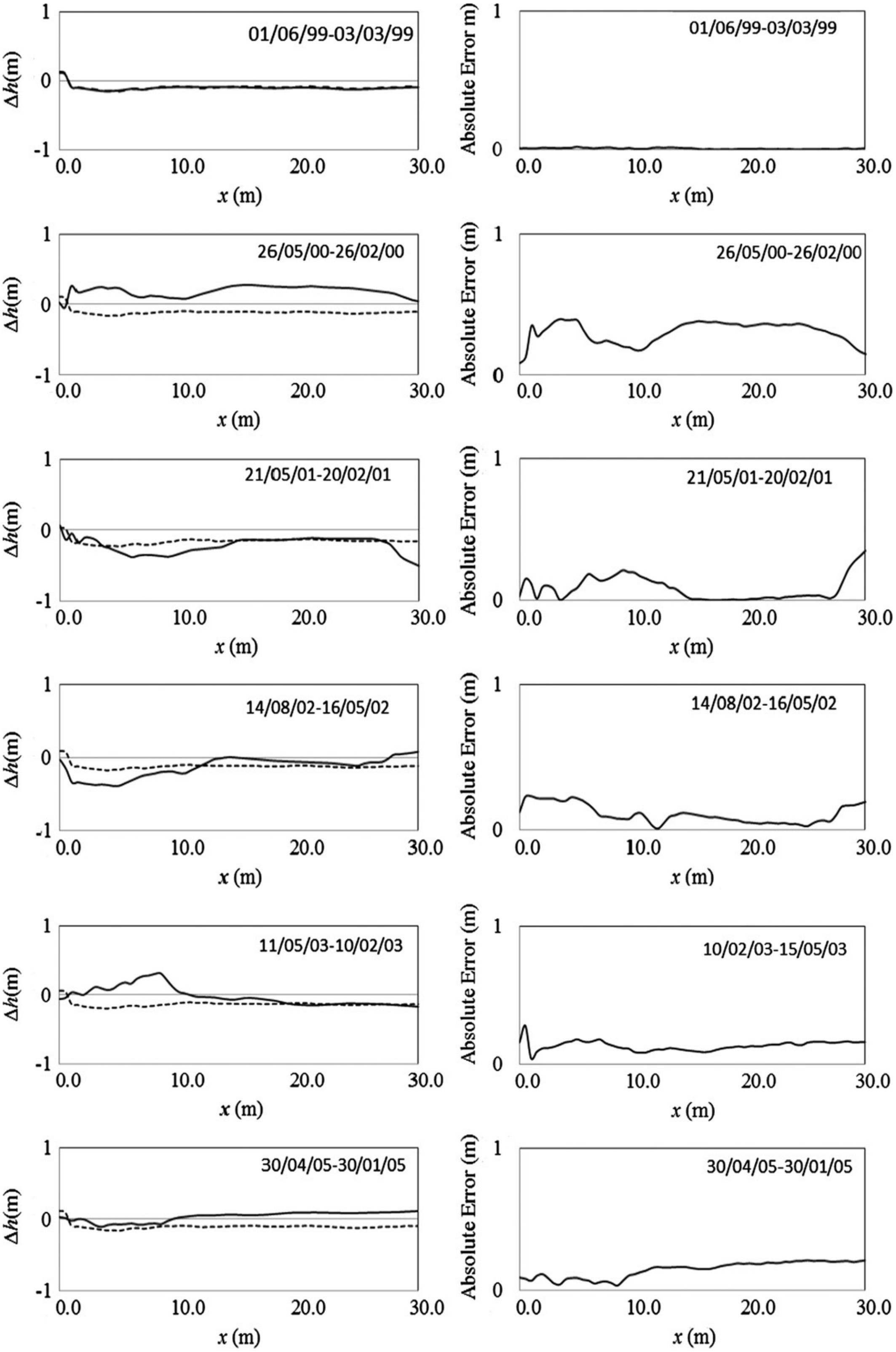

Fig. 12. Comparison of measured (dark line) and computed (broken line) cross-shore profile change by the direct CCA model [left panel] and absolute error between the two [right panel].

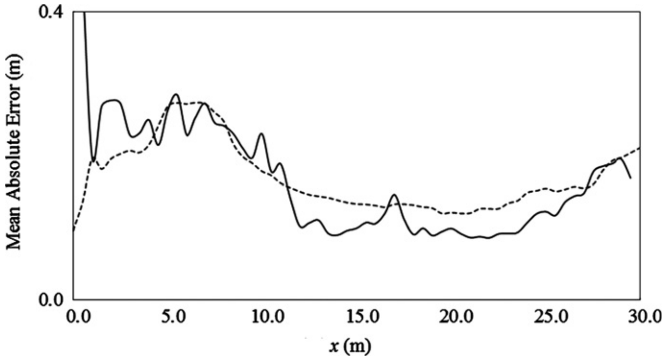

Fig. 13. Mean absolute prediction error for predicted profile changes between 1999 and 2005. Solid line corresponds to diffusion model and the dotted line corresponds to direct CCA approach.

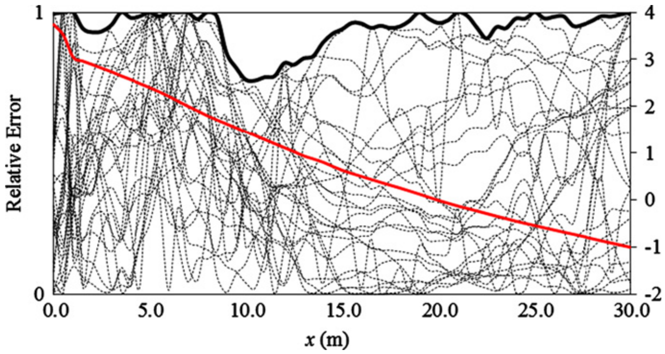

Fig. 14. The relative prediction errors of profile change predictions from the direct CCA approach for all cases from 1999 to 2005. Dark line shows the errors envelop and the red line shows the mean profile in meters. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

diffusion model) are shown in Fig. 14. It can be seen in this figure that the relative prediction errors of the direct CCA approach are uniform across the profile, whereas for the diffusion model they are not suggesting that the source-diffusion mechanism captures a significant amount of the morphological change in the middle of the profile.

5. Conclusions

A cross-shore profile evolution model based on a diffusion type formulation combined with a data-driven approach is presented and demonstrated in this paper. In the diffusion formulation, the space varying diffusion coefficient and a source function that describes the aggregation of all non-diffusive morphodynamic processes govern the success of its ability to predict beach profile evolution in time. This type of model is considered to be a hybrid approach for beach profile modelling.

The model requires historic measurements of beach profiles and waves to determine the unknown parameters. Milford-on-Sea beach, located in the Christchurch Bay beach system in the UK, has been used to demonstrate the model since this site contains cross-shore beach profiles and waves measured over two decades.

The comparison of predicted and measured cross-shore beach profile change, at transect 5f00076 at Milford-on-Sea beach gives encouraging results, despite the simplicity of the modelling approach. It should be noted that the model performance relies on the quality and the availability of the historic beach profile and wave data as key model parameters, the diffusion coefficient and the source function are both derived from the measured data. As implemented here, estimates of the source function used in prediction are made on the basis of the historical correlation between the source function and wave conditions. Thus, when external morphodynamic drivers other than incident waves contribute to the evolution of the source function, discrepancies can arise.

Although not demonstrated here, the model can be used to forecast future changes in beach profiles using forecast wave conditions. The modelling procedure is numerically stable but, since the model is data-dependent, the results are site-specific and the accuracy of model predictions will be dependent upon that of the historical measurements.

Finally, the diffusion model was compared with a direct CCA based data-driven model (Horrillo-Caraballo and Reeve, 2010) that predicts cross-shore beach profiles directly from historic profile surveys and incident wave data. Both models produce encouraging results. However, an explicit inclusion of diffusive processes, together with an aggregated source term leads to improved accuracy in the inter-tidal zone.

Acknowledgement

HK and DER acknowledge support from EPSRC through Grant EP/C005392/1–RF-PeBLE (A Risk-based Framework for Predicting Long-term Beach Erosion). JMH-C and DER acknowledge the support of the European Commission through FP7, 2009–1, Contract 244104–THESEUS (“Innovative technologies for safer European coasts in a changing climate”). The authors also wish to acknowledge New Forest District Council, UK for providing beach profile data.

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