Memo

To
Department EHY

Date
2014-07-21

Number of pages
9

From
Yoeri Dijkstra

E-mail
yoeri.dijkstra@deltares.nl

Subject
Salt balances as diagnostic tools

Salt intrusion in estuaries has been shown to be a difficult modelling problem. The modelling of salt intrusion has been approached in several ways. On the one hand there is the prototype complex model study using 3D models. Although this is a very powerful method, the experience with, for example, the Zeedelta model has shown that such type of models do not always simulate the correct amount of salt intrusion. Such 3D models then lack the tools for analysing why the salt intrusion is modelled incorrectly, or even for understanding what physics governs the salt intrusion.

On the other hand there are the very idealised models such as in the work done by Savenije (overview in Savenije (2005)) or Kuijper and Van Rijn (2011), who use empirical or semi-empirical methods for computing the salt intrusion by using a 1D model. Such studies generally result in a functional dependence of a salt-dispersion coefficient K on estuary dimensions and basic flow properties. Although these methods do not provide the level of detail that one would want for the evaluation of human interventions in the estuary, they do contain a lot of empirical knowledge and understanding of salt intrusion. They therefore have given meaning and interpretation to the dispersion coefficient K.

The aim of this memo is to make a first step in connecting the above two methods by showing how an equivalent dispersion coefficient K can be calculated from the model output of a complex model. This is outlined in Section 1. In Section 2 it will be shown how this dispersion coefficient can be decomposed into several contributions that have a physical interpretation and that can be related to some studies that evaluate measurements of salt intrusion. Next, some remarks are made concerning the advantages, disadvantages and necessary further research in Section 3. The theory in this memo is illustrated in Section 4 by an example that makes use of my own idealised 2DV model in which this approach is already implemented. Finally, the findings are summarised in Section 5.

1 The salt balance

In this section it will be shown how the transport equation for salt relates to the width-averaged depth-averaged time-averaged salt balance. For simplicity of notation all equations are given in rectangular (x, y, z)-coordinates.

The 3D transport equation for salinity reads:

$$\frac{\partial s}{\partial t}+\frac{\partial us}{\partial x}+\frac{\partial vs}{\partial y}+\frac{\partial ws}{\partial z}=\frac{\partial}{\partial x}\left(K_H\frac{\partial s}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_H\frac{\partial s}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_V\frac{\partial s}{\partial z}\right), \quad (1)$$

where s is salinity, u, v and w are velocity components. K_H is a horizontal dispersion coefficient, which in Delft 3D-FLOW is a parameter that depends on horizontal turbulence and a user-defined background diffusion. K_V is a vertical dispersion coefficient that is related to the eddy viscosity by a constant Prandt-Schmidt number.

The 1D counterpart of this salinity equation is

$$\frac{\partial A\overline{s}}{\partial t}+Q\frac{\partial \overline{s}}{\partial x}=\frac{\partial}{\partial x}\left(A\overline{K}\frac{\partial \overline{s}}{\partial x}\right), \quad (2)$$

where the overline $\bar{.}$ indicates a cross-sectional average, A is the cross-sectional area and Q is the river discharge.

The salt balance that will be used as the tool for the analysis is related to this 1D salinity equation. The salt balance computes the time-average salt transport M_t through a cross-section, which simply equals the product of the velocity u and salinity s:

$$M_t=\langle A\overline{us}\rangle, \quad (3)$$

where the brackets $\langle . \rangle$ denote time-averaging.

The averaging procedures that have been introduced need a definition. Firstly, the time-average is the average over some tidal time-scale, which may be either the semi-diurnal tide or a spring-neap cycle, depending on the application. Secondly, the cross-sectional average is defined as the average over the time-average cross-section.

We will use a more general form of the salt balance than Equation 3. This more general form includes the temporal variation of salinity on a time-scale $\hat{t}$ that is larger than the averaging time-scale. It also includes a dispersion term that describes the salt transport that is parametrised by the dispersion coefficient K_H (see Equation 1), which can be regarded as the unexplained salt transport. The general form of the salt balance then reads

$$\frac{\partial}{\partial \hat{t}}\left(\int_x^L A\overline{s}\,dx\right)+\langle A\overline{us}\rangle=A\overline{K}_H\frac{\partial\langle\overline{s}\rangle}{\partial x}, \quad (4)$$

This salt balance can be used directly for the analysis of salt intrusion. However, we would like to make use of the experience that is gained with the dispersion coefficient K in the 1D salt model and so rewrite the salt balance to a form involving an equivalent dispersion coefficient. Let us assume that we can write

$$\langle A\overline{us}\rangle=-A\overline{K}_u\frac{\partial\langle\overline{s}\rangle}{\partial x}.$$

We then find

$$\frac{\partial}{\partial \hat{t}}\left(\int_x^L A\overline{s}\,dx\right)=A\left(\overline{K}_H+\overline{K}_u\right)\frac{\partial\langle\overline{s}\rangle}{\partial x}, \quad (5)$$

1.1 Relation to the 1D salt equation

The 1D salt equation 2 can be related to the salt balance 5. In order to do so we take the time-average of the 1D equation and then integrate this in the x-direction

$$\left\langle \frac{\partial}{\partial t}\left(\int_x^L A\overline{s}\,dx\right)\right\rangle + \left\langle \int_x^L Q\frac{\partial \overline{s}}{\partial x}\,dx\right\rangle = A\overline{K}\frac{\partial \left\langle \overline{s}\right\rangle}{\partial x}$$

This equation already resembles the salt balance. The second term needs some extra attention. If it is assumed that the discharge is constant in time on the averaging time-scale term, then this term is equal to $Q\left\langle \overline{s}\right\rangle$. This term is a part of the term $\left\langle A\overline{us}\right\rangle$ in the salt balance as will be clear from the decomposition in Section 2.

The 1D salt equation can then be related to Equation 5 by writing $Q\left\langle \overline{s}\right\rangle$ as a dispersion term according to

$$\left\langle \int_x^L Q\frac{\partial \overline{s}}{\partial x}\,dx\right\rangle = -AK_Q\frac{\partial \langle \overline{s}\rangle}{\partial x}.$$

The relation between the two equations can then simply be expressed by dispersion coefficient according to

$$K = K_H + K_u - K_Q.$$

2 Decomposition technique

The salt transport can be decomposed into several terms. The approach that will be used here is the approach by Fischer (1972). First, we make the following decomposition of the velocity and salinity

$$u = u_a + u_b + u_c + u_d, \tag{6}$$

$u_a = \langle \overline{u}\rangle$	cross-sectionally-averaged time-averaged velocity
$u_b = \overline{u} - u_a$	cross-sectionally-averaged time-varying velocity
$u_c = \langle u\rangle - u_a$	cross-sectional variation of the time-averaged velocity
$u_d = u - u_a - u_b - u_c$	cross-sectional variation of the time-varying velocity
$\quad = u - \langle u\rangle - \overline{u} + \langle \overline{u}\rangle$	

For salinity a similar decomposition is made.

The terms u_c, s_c, u_d and s_d can be divided in a vertically varying part, denoted by subscript z, and a laterally varying part, denoted by subscript y.

This decomposition can now be used to find a decomposition of the explained mass transport $\langle A\overline{us}\rangle$. The full decomposition results in an equation containing 16 terms. In order to simplify this equation, some of these terms are put together in one term with comparable phyisical meaning. The resulting equation is

$$M_t(x) = Qs_a + A\left\langle u_b s_b\right\rangle^* + A\overline{u_{c,z}s_{c,z}}^* + A\overline{u_{c,y}s_{c,y}}^* + A\left\langle \overline{u_{d,z}s_{d,z}}\right\rangle^* + A\left\langle \overline{u_{d,y}s_{d,y}}\right\rangle^* \tag{7}$$

The asterisks * denote that the term is not exactly equal to the term you might expect from the decomposition, but that it is corrected for the Stokes drift. Appendix A provides the details of these terms.

2.1 Meaning of the terms

The terms in the above equations have the following meaning. $s_a Q$ is the total advection of salt by the river (or other) discharge. This term is easy to interpret and taken into account by all authors that consider salt transport. It is also the most certain term, because the discharge is prescribed by the user. It can therefore be used as a reference to compare other terms with.

The component $\langle u_b s_b \rangle$ could be called *tidal oscillatory uniform dispersion*. It is the transport due to the interaction of the cross-sectionally-averaged tidal velocity and salinity, which can potentially yield a large contribution. In literature this term is generally either neglected (e.g. Fischer (1972)) or used together with the terms containing u_d (e.g. Lerczak et al. (2006)). However, this is, in my opinion, unjustly done so. Fischer (1972) reasons that the term is negligible in well-mixed to partially stratified estuaries, because s_b is much smaller than s_a. However, u_b (the typical tidal velocity) is generally much larger than u_a (the typical residual velocity). It will be shown that Fischer's reasoning is wrong in the example of Section 4. It is useful to consider this term separately from the terms containing u_d, because it is generally known from measurements what the typical magnitudes of the average tidal velocity and tidal variation of salinity are. The magnitude of $\langle u_b s_b \rangle$ can thus be estimated from only a few measurements. On the other hand, the variations of velocity and salinity over the cross-section are often unknown.

It is particularly interesting to look at the phase of u_b compared to the phase of s_b. It will be shown in the example of Section 4 that a few degrees phase difference already causes a great change in the salt transport. The term is therefore also a measure of this phase difference.

The terms $\overline{u_{c,z} s_{c,z}}$ and $\overline{u_{c,y} s_{c,y}}$ are the *vertical and lateral steady shear dispersion* (Taylor, 1953), also known as transport due to estuarine circulation. This contains most notably the transport by gravitational circulation and straining circulation in longitudinal and lateral direction. The lateral component additionally contains the net lateral circulation which is due to curvature of the estuary or tidal pumping (e.g. ebb-flood channels).

Finally the terms $\langle \overline{u_{d,z} s_{d,z}} \rangle$ and $\langle \overline{u_{d,y} s_{d,y}} \rangle$ are the *vertical and lateral tidal oscillatory shear dispersion*. They denote the net transport of salt due to shearing of the velocity and salinity profiles. These terms directly reflect many subtle properties of the flow. For example the vertical term depends on the degree and the timing of vertical stratification and the shape of the vertical velocity profile.

2.2 Studies using measurements

Since the decomposition by Fischer (1972), these terms have been identified in several studies on measurements of salt intrusion. Some examples will be given here. The study done by Lerczak et al. (2006) relates measurements in the Hudson River to three quantities: $s_a Q$, $\overline{u_c s_c}$ (in their notation $\mathcal{F}_\epsilon$) and the combination $\langle u_b s_b + \overline{u_d s_d} \rangle$ (in their notation $\mathcal{F}_\tau$). They have not made a distinction between lateral and vertical effects. A similar study was done earlier by Jay and Smith (1990) on the Columbia River.

Winterwerp (1983) related measurements of the Rotterdam Waterway and a WL flume experiment to the components of this salt balance, but disregarded lateral variations. He used the same decomposition and gave the magnitude of all the terms in this balance. He also considered the salt balance without time-averaging in order to investigate the driving mechanism behind the tidal movement of the salt wedge in the Rotterdam Waterway.

3 Advantages, disadvantages and possibilities

The salt balance has been used with success in many studies, but is also criticised in literature. Some of the considerations will be given below.

Use of the salt balance

Let us start with a criticism. Savenije (2005) has included a section with the bold title *"the decomposition method and why it is not very useful"*. His reasoning is, however, more subtle and can be summarised by saying that the decomposition is not useful, because it is easily used in the wrong way. In order to illustrate this, we use a quote of Chatwin and Allen (1985): *"the question of whether the transverse or the vertical dispersion is the most important salt intrusion mechanism may be less fundamental than was once believed"*. This quote reflects that it is rather artificial to decompose the dispersion in separate terms, among which separate vertical and lateral processes, as if they can be seen independently. The opposite is true; the terms from strong interactions between the processes so that, for example, removing all lateral processes from the model will also yield a very different vertical dispersion. One should therefore not regard the salt balance as a decoupling into independent or weakly dependent components.

What the salt balance does provide is a framework of thinking about the processes that govern salt intrusion and a way of comparing models, measurements and theory. It is thus a tool that requires the interpretation of a skilled user of the model.

Decomposition in physical components

Some of the terms in the decomposed salt balance 7 are composed of several different physical mechanisms. An example is the term $\langle \overline{u_{d,y} s_{d,y}} \rangle$, which contains the effect of lateral gravitational circulation, lateral straining circulation, lateral secondary circulation (due to curvature of the channel) and tidal pumping (e.g. due to ebb-flood channels). It is hard or even impossible to unambiguously separate these components in the output of a complex model.

It might, however, be possible to visualise the bathymetric effects (i.e. tidal pumping) by investigating the salt balance without width-averaging. If the estuary is dominated by channel-shoal patterns or multiple branches this might be a good alternative. This possibility has not been researched yet.

Open for research

Some of the significant physical effects that were not mentioned yet are tidal trapping and the complex physics of salt inflow at the mouth of the estuary. Tidal trapping is the storage and release of salt water in dead-end branches such as harbour basins. The exchange between the main channel and the dead-end basin may have a different phasing than the phasing of the tidal flow in the main channel. This could result in net contributions to mixing, residual flows and salt transport. It is not yet investigated in what terms of the decomposed salt balance tidal pumping enters.

4 Example with idealised model code

The use of the salt balance will be illustrated by a simulation of the idealised model that I developed for my thesis. This idealised model is a 2DV model of a funnel-shaped estuary with a rectangular cross-section. It solves the continuity, momentum and salinity equations. The model contains several options for turbulence closures, among which the $k - \epsilon$ model. The example below uses the $k - \epsilon$ turbulence model. The example uses a model of an estuary with the dimensions given in Table 1.

Parameter	Symbol	Value
Length	L	100 km
Width at mouth	B_0	2500 m
Depth	H_0	25 m
Convergence length	L_b	30 km
M_2 tidal amplitude at mouth	A_{M_2}	2.0 m
River discharge	Q	100 m^3/s

Table 1: Parameter values for estuary dimensions and forcing.

The model is a 2DV model does not resolve the lateral contribution to salt intrusion. This is therefore parametrised by using a constant transport $K_H = 100$ m^2/s plus some correction to close the salt balance. This correction is mainly due to the approximations in the idealised model and will not appear if the method is used for analysing Delft 3D output.

Figure 1 shows the salt balance. The blue line (with the large negative value) depicts the equivalent dispersion coefficient that can be attributed to the river discharge. The total value of the dispersion coefficient in Equation 2 equals -1 times this river induced dispersion. The figure shows that the value of K is not constant in the estuary. This reflects that the shape of the salt intrusion curve is not a simple exponential such as would be produced if the salinity equation were a pure diffusion equation; the gradient of K introduces an advection part in Equation 2.

The value of $\langle u_b s_b \rangle$ yields the most important model resolved contribution to salt intrusion. A great advantage of the term $\langle u_b s_b \rangle$ over many of the other terms is that it is relatively easy to analyse; its magnitude can be estimated from three variables: the cross-sectionally-averaged tidal velocity and cross-sectionally-averaged tidally-varying salinity and their phase difference. The first two of these variables can often be estimated from measurements or from the model output (e.g. in Quickplot). The term $\langle u_b s_b \rangle$ therefore puts some emphasis on the importance of the phase difference between velocity and salinity. Early results with my idealised model have shown that such phase difference is especially sensitive to the turbulence model.

The value of $\overline{u_c s_c}$ remains small, because the subtidal salinity field is well mixed ($s_c \approx 0$). It follows that the direct contribution of the gravitational circulation and straining circulation (i.e. the most important contributions to u_c) are unimportant to the salt intrusion. This does not mean that the gravitational circulation and straining circulation are unimportant. They might still have an indirect effect on salt intrusion by altering, for example, the level of turbulence and therefore all flow velocities.

The case under consideration is a well-mixed estuary (i.e. s_d small) so that $\langle \overline{u_d s_d} \rangle$ is small. This term becomes much more important if stratification is stronger.

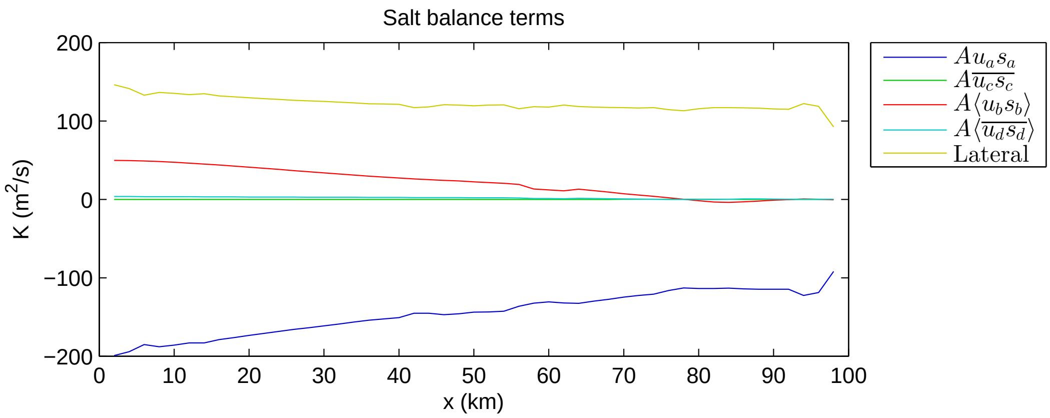

Figure 1: Terms in the salt balance expressed in a dispersion coefficient. Case 1: eddy viscosity almost constant. The bigger terms are shown in the left figure and the smaller terms are shown in the right figure.

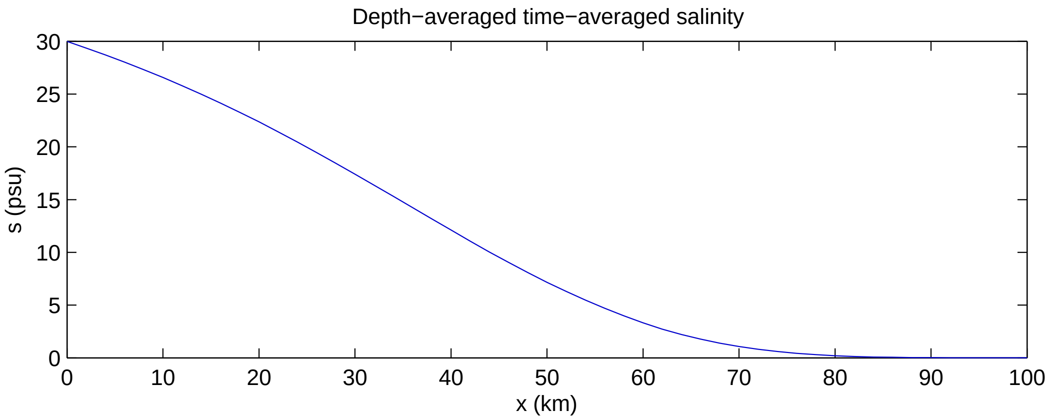

Figure 2: Along-channel evolution of the depth-averaged time-averaged salinity.

5 Summary

It was shown in this memo that it is possible to represent the salt-intrusion modelled by a 2DV or 3D model by using the salt-dispersion coefficient that is well known for 1D salt models. This provides a new way of interpreting the output of 2DV or 3D models with the empirical knowledge that has been obtained with 1D models. The dispersion coefficient has been decomposed in six terms that have a physical interpretation and that have been used in several studies that discuss the salt intrusion in estuaries on the basis of measurements.

The six terms in the mass balance depend strongly on one-another and should not be regarded as independent terms. The decomposition is, however, useful because it provides a framework for thinking about the processes that govern salt intrusion in the estuary.

Further research is needed to investigate if it is possible to distinguish bathymetric effects (e.g. ebb and flood channels) and the effects of side channels (e.g. dead-end basins connected to the estuary) in the salt balance.

References

Chatwin, P. C. and Allen, C. M. (1985). Mathematical models of dispersion in rivers and estuaries. *Annual Review of Fluid Mechanics*, 17:119–149. 5

Fischer, H. (1972). Mass-transport mechanisms in partially stratified estuaries. *J. Fluid Mech.*, 53:671–687. 3, 4

Jay, D. A. and Smith, J. D. (1990). Circulation, density distribution and neap-spring transitions in the columbia river estuary. *Progress in Oceanography*, 25(14):81 – 112. 4

Kuijper, K. and Van Rijn, L. C. (2011). Analytical and numerical analysis of tides and salinities in estuaries; part II: salinity distributions in prismatic and convergent tidal channels. *Ocean Dynamics*, 61(11):1743–1765. 1

Lerczak, J. A., Geyer, W. R., and Chant, R. J. (2006). Mechanisms driving the time-dependent salt flux in a partially stratified estuary. *J. Phys. Oceanogr.*, 36(12):2296–2311. 4

Savenije, H. (2005). *Salinity and tides in alluvial estuaries*. Elsevier, Amsterdam. 1, 5

Taylor, G. (1953). Dispersion of soluble matter in solvent flowing slowly through a tube. *Proceedings of the Royal Society of London*, A219:186–203. 4

Winterwerp, J. C. (1983). Decomposition of the mass transport in narrow estuaries. *Estuarine Coastal and Shelf Science*, 16:627–638. 4

A Full salt balance decomposition

The full decomposition of the salt flux $\langle\overline{us}\rangle$ using the above decomposition contains 16 terms and reads

$$\begin{aligned} M_t(x) =& Au_a s_a + s_a\langle A' u_b\rangle + s_a\langle A' u_d(0)\rangle \\ &+ A\overline{u_c s_c} + s_c(0)\langle A' u_b\rangle + s_c(0)\langle A' u_d(0)\rangle \\ &+ A\langle u_b s_b\rangle + u_a\langle A' s_b\rangle + u_c(0)\langle A' s_b\rangle + \langle A' u_b s_b\rangle + \langle A' u_d(0) s_b\rangle \\ &+ A\langle\overline{u_d s_d}\rangle + u_a\langle A' s_d(0)\rangle + u_c(0)\langle A' s_d(0)\rangle + \langle A' u_d(0) s_d(0)\rangle + \langle A' u_b s_d(0)\rangle. \end{aligned} \tag{8}$$

Recall that, in this equation, A is defined as the time averaged cross-sectional area, i.e. $A = \langle A\rangle$. The temporal deviation of A is denoted by A'.

Many of these represent a Stokes drift transport. It is chosen to combine the Stokes drift terms with the depth-averaged term that contains the same salinity component, i.e.

$$\begin{aligned} Qs_a &= Au_a s_a + s_a\langle\zeta u_b\rangle + s_a\langle\zeta u_d(0)\rangle, \\ A\langle u_b s_b\rangle^* &= A\langle u_b s_b\rangle + u_a\langle A' s_b\rangle + u_c(0)\langle A' s_b\rangle + \langle A' u_b s_b\rangle + \langle A' u_d(0) s_b\rangle, \\ A\overline{u_c s_c}^* &= A\overline{u_c s_c} + s_c(0)\langle A' u_b\rangle + s_c(0)\langle A' u_d(0)\rangle, \\ \langle\overline{u_d s_d}\rangle^* &= A\langle\overline{u_d s_d}\rangle + u_a\langle A' s_d(0)\rangle + u_c(0)\langle A' s_d(0)\rangle + \langle A' u_d(0) s_d(0)\rangle + \langle A' u_b s_d(0)\rangle \end{aligned}$$

The idea behind this grouping stems from the first line, in which all terms involving s_a are grouped. The terms on this line have a clear meaning. Let us consider the situation of homogeneous flow and let us replace s by a constant density ρ, then the mass reduces to

$$M_t(x) = Au_a\rho + \rho\langle A'u_b\rangle + \rho\langle A'u_d(0)\rangle.$$

This equation equals the water balance. The last two terms are Stokes drift terms. These Stokes drift terms are compensated for by a return flow that is contained within u_a. The net result should be equal to the river discharge $Q\rho$, so it is physically meaningful to group terms that contain s_a. Similar arguments do not exist for grouping the other terms, because the remaining Stokes drift terms do not vanish.