Annex E: Input and output of the Artificial Limit State function

Annex E.1: Standard linear Resistance-Solicitation problem

Limit state function:

$g = R-S$

Table: Input variables

Variable	μ	σ	
R	7.0	1.0	N
S	2.0	1.0	N

Table: Results

Method	β	LSFE	$V(P_f)$
Exact	3.54		-
FORM basic	3.54	6	-
FORM NLPQL	3.54	7	-
FORM RFLS	3.54	9	-
FORM RFLS imp. samp	3.54	309	-
FORM-ARS (u-space)	3.54	6	-
FORM-ARS (x-space)	3.54	6	-
SORM	3.54	12	-
SORM imp. samp.	3.54	314	-
MC (adsamp)	3.54	10000	0.02
MCI(σ_h=3)	3.57	122	0.49
DS	3.55	100	0.49
DARS	3.55	18	0.49

Table: Intermediate results from DARS

Method	ADI	DARS
β	4.62	3.55
λ_{min}	5.00	3.54
$\beta_{RS,FORM}$	3.53	3.54

Annex E.2: Noisy limit state function

Limit state function:

$$g = x_1 + 2x_2 + 2x_3 + x_4 - 5x_5 - 5x_6 + 0.001\sum_{i=1}^{6}\sin(100x_i)$$

Table: Input variables

Variable	μ	σ	
x_1	120	12	LN
x_2	120	12	LN
x_3	120	12	LN
x_4	120	12	LN
x_5	50	15	LN
x_6	40	12	LN

Table: Results

Method	β	LSFE	$V(P_f)$
Exact [Eng93]	2.25		-
FORM basic	2.35	40	-
FORM NLPQL	2.35	44	-
FORM RFLS	2.35	452	-
FORM RFLS imp. samp.	2.23	1010	-
FORM-ARS (u-space)	2.35	37	-
FORM-ARS (x-space)	2.35	15	-
SORM	2.25	71	-
SORM imp. samp.	2.26	973	-
MC (adsamp)	2.24	10000	0.01
MCI(σ_h=2)	2.24	1225	0.25
DS	2.24	925	0.25
DARS	2.24	271	0.25

Table: Intermediate results from DARS

Method	ADI	DARS
β	1.75	2.24
λ_{min}	2.67	2.54
$\beta_{RS,FORM}$	2.09	2.19

Annex E.3: A Resistance-Solicitation problem with one quadratic term

Limit state function:

$$g = R\text{-}S^2$$

Table: Input variables

Variable	μ	σ	
R	11.0	1.0	N
S	1.5	0.5	N

Table: Results

Method	β	LSFE	$V(P_f)$
Exact	3.46		-
FORM basic	3.47	12	-
FORM NLPQL	3.47	19	-
FORM RFLS	3.47	14	-
FORM RFLS imp.samp	3.47	346	-
FORM-ARS (u-space)	3.47	7	-
FORM-ARS (x-space)	3.47	6	-
SORM	3.46	24	-
SORM imp.samp	3.47	319	-
MC (adsamp)	3.47	10000	0.02
MCI(σ_h=3)	3.50	267	0.49
DS	3.43	60	0.49
DARS	3.43	38	0.49

Table: Intermediate results from DARS

Method	ADI	DARS
β	3.40	3.43
λ_{min}	3.63	3.48
$\beta_{RS,FORM}$	3.53	3.76

Annex E.4: Limit state function with 10 quadratic terms

Limit state function:

$$g = R - \sum_{i=1}^{10} S_i^2/i$$

Table: Input variables

Variable	μ	σ	
R	0.5	0.1	N
S_i	0.2	0.1	N

Table: Results

Method	β	LSFE	$V(P_f)$
Exact	2.98	-	-
FORM basic	3.20	65	-
FORM NLPQL	3.20	61	-
FORM RFLS	3.20	74	-
FORM RFLS imp. samp	2.98	574	-
FORM-ARS (u-space)	3.20	14	-
FORM-ARS (x-space)	3.20	16	-
SORM	2.97	151	-
SORM imp. samp.	2.96	637	-
MC (adsamp)	2.98	10000	0.11
MCI(σ_h=2)	3.08	2868	0.39
DS	2.90	1132	0.39
DARS	2.90	221	0.39

Table: Intermediate results from DARS

Method	ADI	DARS
β	2.29	2.90
λ_{min}	3.82	3.64
$\beta_{RS,FORM}$	2.67	3.52

Annex E.5: Limit state function with 25 quadratic terms

Limit state function:

$$g = R - \sum_{i=1}^{25} S_i^2/i$$

Table: Input variables

Variable	μ	σ	
R	0.5	0.1	N
S_i	0.2	0.1	N

Table: Results

Method	β	LSFE	$V(P_f)$
Exact	2.63	-	-
FORM basic	2.92	84	-
FORM NLPQL	2.84	136	-
FORM RFLS	2.84	164	-
FORM RFLS imp. samp.	2.52	674	-
FORM-ARS (u-space)	2.94	57	-
FORM-ARS (x-space)	2.94	57	-
SORM	2.52	541	-
SORM imp. samp.	2.63	314	-
MC (adsamp)	2.98	10000	0.27
MC (crude)	2.63	100000	-
MCI(σ_h=1.5)	2.74	6501	0.31
DS	2.65	2540	0.31
DARS	2.65	188	0.31

Table: Intermediate results from DARS

Method	ADI	DARS
β	1.79	2.65
λ_{min}	3.47	3.47
$\beta_{RS,FORM}$	2.33	3.17

Annex E.6: Convex failure domain

Limit state function:

$$g = 0.1(u_1 - u_2)^2 - (u_1 + u_2)/\sqrt{2} + 2.5$$

Both variables u_1 and u_2 have a standard normal distribution.

Table: Results

Method	β	LSFE	$V(P_f)$
Exact	2.63	-	-
FORM standard	2.50	8	-
FORM NLPQL	2.50	7	-
FORM RFLS	2.50	9	-
FORM RFLS imp. samp.	2.61	484	
FORM-ARS	2.50	7	-
SORM	2.63	14	-
SORM imp. samp.	2.63	314	-
MC (adsamp)	2.63	10000	0.02
MCI(σ_h=1.5)	2.63	513	0.32
DS	2.63	208	0.32
DARS	2.61	47	0.32

Table: Intermediate results from DARS

Method	ADI	DARS
β	∞	2.61
λ_{min}	∞	2.50
$\beta_{RS,FORM}$	2.88	2.50

Annex E.7: Oblate spheroid

Limit state function:

$$g = R - \sum_{i=1}^{10} S_i^2/(1+i/10)$$

with R = 10 and S_i is standard normal distributed.

Table: Results

Method	β	LSFE	$V(P_f)$
Exact	1.10	-	-
FORM standard	3.33	204	-
FORM NLPQL	3.31	187	-
FORM RFLS	3.31	482	-
FORM RFLS imp. samp.	1.93	1319	-
FORM-ARS	3.21	22	-
SORM	*	-	-
SORM imp. samp.**	0.97	191613	-
MC (adsamp)	1.26	100000	0.15
MC (crude)	1.10	100000	-
MCI(σ_h=2)	1.10	1682	0.09
DS	1.07	170	0.09
DARS	1.06	160	0.09

** error message*

*** 10.000 imp. samp used for convergence to $V(P_f) < 0.08$*

Error message of SORM

Warning from 2nd-order improvement:
There are very large curvatures at u-.*
2nd-order improvement might be biased !
1st-order result is likely to have a large error !

Table: Intermediate results from DARS

Method	ADI	DARS
β	1.05	1.06
λ_{min}	3.32	3.32
$\beta_{RS,FORM}$	46.4	46.4

Annex E.8: Saddle surface

Limit state function:

$$g = 3 - u_1 u_2$$

Both variables are standard normal distributed.

Table: Results

Method	β	LSFE	$V(P_f)$
Exact	2.34	-	-
FORM standard*	2.86	16	-
FORM NLPQL*	2.45	47	-
FORM RFLS*	2.45	36	-
FORM RFLS imp. samp.*	2.56	620	-
FORM-ARS*	2.50	33	-
SORM*	2.58	41	-
SORM imp. samp.*	2.58	409	-
MC (adsamp)	2.58	10000	-
MC (crude)	2.32	100000	-
MCI(σ_h=3)	2.34	385	-
DS	2.26	299	0.25
DARS	2.35	225	0.25

**FORM/SORM starting in point (0,0) results in an error message*

Table: Intermediate results from DARS

Method	ADI	DARS
β	∞	2.35
λ_{min}	∞	2.45
$\beta_{RS,FORM}$	∞	0.0

Annex E.9: Discontinuous limit state function

Limit state function:

$$g = -0.5 + \sqrt{R - S} \cdots \forall R \geq S$$
$$g = -0.5 \cdots\cdots\cdots \forall R < S$$

Table: Input variables

Variable	μ	σ	
R	15.0	2.5	N
S	5.0	0.5	N

Table: Results

Method	β	LSFE	$V(P_f)$
Exact	3.83	-	-
FORM standard	*	-	-
FORM NLPQL	6.60	7	-
FORM RFLS	*	-	-
FORM RFLS imp. samp.	*		
FORM-ARS (u-space)	3.85	11	-
FORM-ARS (x-space)	3.84	13	-
SORM	6.60	12	-
SORM imp. samp.	*		
MC (adsamp)	3.82	10000	0.02
MC (crude)	3.83	1000000	-
MCI(σ_h=2)	3.84	325	0.54
DS	3.94	333	0.54
DARS	3.84	55	0.54

** error message*

Table: Intermediate results from DARS

Method	ADI	DARS
β	3.66	3.84
λ_{min}	3.90	3.83
$\beta_{RS,FORM}$	3.83	3.82

Annex E.10: Two branches

Limit state function:

$$g = x_1 - x_2 - x_3 \cdot \cdot \forall x_3 \leq 5$$
$$g = x_3 - x_2 \cdot \cdot \cdot \cdot \cdot \cdot \cdot \forall x_3 > 5$$

Table: Input variables

Variable	μ	σ	
x_1	10.0	0.5	N
x_2	0.0	1.0	N
x_3	4.0	1.0	N

Table: Results

Method	β	LSFE	$V(P_f)$
Exact	5.03	-	-
FORM standard	-	-	-
FORM NLPQL	4.00	244	-
FORM RFLS	2.21*	769	-
FORM RFLS imp. samp.	*		
FORM-ARS (u-space)	*	-	-
FORM-ARS (x-space)	4.84	29	-
SORM	4.51	253	-
SORM imp. samp.	*		
MC (adsamp)	5.03	10000	0.06
MCI(σ_h=3)	5.15	2787	0.57
DS	5.01	728	0.57
DARS	5.00	135	0.57

* *error message*

Table: Intermediate results from DARS

Method	ADI	DARS
β	5.57	5.00
λ_{min}	6.00	4.70
$\beta_{RS,FORM}$	5.37	4.71

Annex E.11: Concave failure domain

Limit state function:

$$g = -0.5(u_1^2 + u_2^2 - 2u_1u_2) - (u_1 + u_2)/\sqrt{2} + 3.0$$

Both variables have a standard normal distribution.

Table: Results

Method	β	LSFE	$V(P_f)$
Exact	1.26	-	-
FORM standard	1.66	8	-
FORM NLPQL	1.66*	7	-
FORM RFLS	1.66*	9	-
FORM RFLS imp. samp.	1.64	407	-
FORM-ARS	2.06	18	-
SORM	1.66*	12	-
SORM imp. samp.	1.52	494	-
MC (adsamp)	1.12	10000	0.19
MC (crude)	1.26	100000	-
MCI(σ_h=3)	1.25	1172	0.10
DS	1.27	260	0.10
DARS	1.27	240	0.10

**FORM/SORM starting in point (0,0) results in $\beta = 3.00$*

Table: Intermediate results from DARS

Method	ADI	DARS
β	1.31	1.27
λ_{min}	1.84	1.66
$\beta_{RS,FORM}$	1.71	1.80

Annex E.12: Series system

Limit state function:

$$g_1 = 0.1(u_1 - u_2)^2 - (u_1 + u_2)/\sqrt{2} + 3.0$$
$$g_2 = 0.1(u_1 - u_2)^2 + (u_1 + u_2)/\sqrt{2} + 3.0$$
$$g_3 = u_1 - u_2 + 3.5\sqrt{2}$$
$$g_4 = -u_1 + u_2 + 3.5\sqrt{2}$$
$$g = \min\{g_1, g_2, g_3, g_4\}$$

Both variables have a standard normal distribution.

Table: Results

Method	β	LSFE	$V(P_f)$
Exact	2.85	-	-
FORM standard	3.50	32	-
FORM NLPQL	3.00	7	-
FORM RFLS	3.00	8	-
FORM RFLS imp. samp.	3.11	494	-
FORM-ARS	3.01	8	-
SORM	3.13	12	-
SORM imp. samp.	3.13	313	-
MC (adsamp)	3.04	10000	0.23
MC (crude)	2.85	100000	-
MCI(σ_h=3)	2.87	166	0.37
DS	2.84	227	0.37
DARS	2.92	175	0.37

Table: Intermediate results from DARS

Method	ADI	DARS
β	4.28	2.92
λ_{min}	4.81	3.00
$\beta_{RS,FORM}$	14.5	5.03

Annex E.13: Parallel system

Limit state function:

$g_1 = 2.677 - u_1 - u_2$
$g_2 = 2.500 - u_2 - u_3$
$g_3 = 2.323 - u_3 - u_4$
$g_4 = 2.250 - u_4 - u_5$
$g = \max(g_i)$

All variables have a standard normal distribution.

Table: Results

Method	β	LSFE	$V(P_f)$
exact	3.52		-
FORM standard	*		-
FORM NLPQ	1.85	1086	-
FORM RFLS	*		-
FORM RFLS imp. samp.	*		-
FORM-ARS	*		-
SORM	3.73	523	-
SORM imp. samp.	*		-
MC (adsamp)	3.52	10000	0.05
MC (crude)	3.55	100000	-
MCI(σ_h=3)	3.58	2778	0.49
DS	3.52	581	0.49
DARS	3.52	127	0.49

** error message*

Table: Intermediate results from DARS

Method	ADI	DARS
β	∞	3.52
λ_{min}	∞	3.49
$\beta_{RS,FORM}$	3.25	3.86