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Predicting coastal erosion trends using non-stationary statistics and process-based models

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ABSTRACT

Storms and water levels are subject to seasonal variations but may also have decadal or longer trends that need to be included when estimating risks in the coastal zone. We propose a non-stationary multivariate generalised extreme value model for wave height, wave period, storm duration and water levels that is constructed using Archimedean copulas. The statistical model was applied to a South African case study to test the impacts of decadal trends on beach erosion. Erosion was estimated using three process-based models — SBEACH, XBEACH, and the Time Convolution model. The XBEACH model provided the best calibration results and was used to simulate potential future long-term trends in beach erosion. Based on the simulated erosion results of 5 beach profiles for storms with 25, 50 and 100 year return periods, it is estimated that the erosion rate could increase by 0.20%/year/storm and should therefore be a significant factor in long-term planning.

1. Introduction

Increasing awareness of future climate change impacts has added a new dimension to traditional design practice. The predicted and/or the measured increases in storm intensity and frequency should be accounted for in failure risk assessment based on an average recurrence interval. Erosion of coastlines is dominated by three factors: sediment supply; wave forces and sea level rise. This paper attempts to consider all these factors and forecast the erosion potential of future storms using a non-stationary multivariate generalised extreme value statistical model based on Archimedean copulas together with process-based models of the beach response.

Numerous authors have proposed a combination of process-based models and statistical models to estimate the potential impacts of climate trends. Only the most relevant examples are mentioned here. Wang et al. (2004) analysed potential changes in significant wave heights using a global climate model and a non-stationary generalised extreme value distribution. They concluded that there was variability of about 20% between decadal extreme significant wave heights. Coles and Tawn (1990, 1991, 1994) provide methods relating to multivariate statistical modelling in a coastal context while Coles and Tawn (1994) used these methods with an empirical formula for overtopping of a seawall to estimate a probability zone of failure. Wang and Reeve (2010) presented a probabilistic model of long-term beach evolution near detached breakwaters using the numerical model developed by Hanson et al. (2006). Callaghan et al. (2008) used a joint distribution of wave parameters to estimate erosion in combination with the time convolution shoreline response model of Kriebel and Dean (1993). Zacharioudaki and Reeve (2011) performed a statistical analysis of beach response to wave conditions arising from climate change scenarios. Zacharioudaki and Reeve (2011) used a one-line beach response model which is appropriate for beaches dominated by long-shore sediment transport. Our study is concerned with storm waves and so uses cross-shore morphological models. Although much work has combined statistical models with numerical models this paper presents a unique use of a copula based non-stationary multivariate statistical model in combination with process-based models to quantify potential future storm induced erosion.

We initially provide a brief theoretical background to the statistical and process-based models and outline the methods used. The methodology is tested by applying it to a case study on the east coast of South Africa. The results are then presented and discussed before concluding.

2. Theoretical background and methods

2.1. Case study site

The east coast of South Africa has 18 years of reliable wave data from wave recording buoys near the city of Durban (Fig. 1). Corbella and Stretch (in press) provide details of the data set. A storm event was defined in terms of a significant wave height threshold similar to the triangular storm concept proposed by Boccotti (2000): a storm event begins when a significant wave height *Hs* exceeds a threshold of 3.5 m and ends when the significant wave height falls below 3.5 m for a period of at least 2 weeks based on the decay time of the autocorrelation. The

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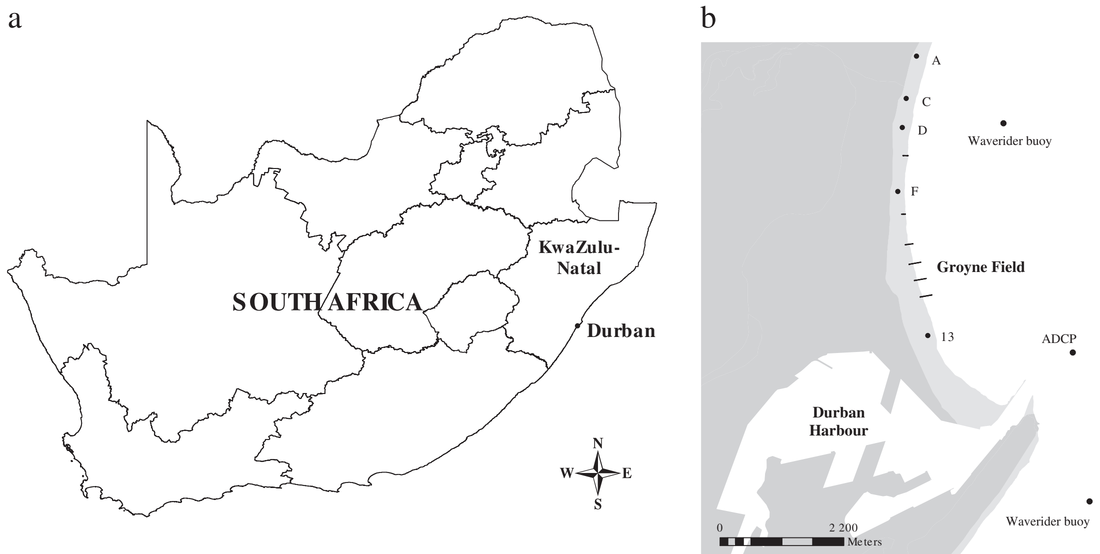

Fig. 1. A map of (a) South Africa showing Durban and a map of (b) the Durban Bight showing the locations of profile A, C, D, F and 13 and the Durban harbour.

reduced storm data set was then manually assessed to ensure that each storm event represented one meteorological event. Finally we selected the largest storms from the remaining group — the number chosen corresponded to an overall average of 3 storms per year. The period between the start and the end time is the storm duration *D* and the time between the events is the calm period *I*. The storm definition is illustrated in Fig. 2.

Corbella and Stretch (2011a) analysed Durban's wave data and identified increasing trends in significant wave heights exceeding the 3.5 m threshold. They also noted an increase in peak period *T* and in the frequency of storm events (or similarly a decrease in the average calm period). Only the increase in peak period was found to be statistically significant.

The case study site at Durban also has a 37 year record of beach profiles which exhibit a long term erosion trend (Corbella and Stretch, 2011a). The records of interest to this study are those that bound storm events. Only the 1998 and 2007 events met these requirements. The analysis is limited to profiles A, C, D, F and 13 (Fig. 1) as they have the most frequent bathymetry data and provide a good representation of the Durban Bight while avoiding most of the sheltering near the harbour entrance and the influence of perpendicular beach structures and sand bypass scheme.

In March 2007 Durban experienced its largest wave event on record. The 8.5 m significant wave height and 16.6 second peak period coincided with an extreme high tide of 2.2 m above chart

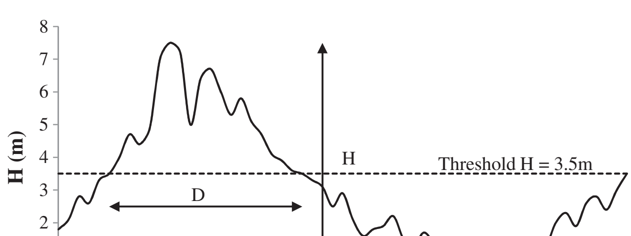

Fig. 2. Illustration of the storm definition showing the significant wave height *Hs*, storm duration *D* and calm period *I*.

datum[1] (CD) and devastated the coastline. This storm was a realisation that much of the current infrastructure is not capable of withstanding potentially more frequent and intense events in the future. Since the damage of the 2007 event can be easily quantified it will be used as a base line to demonstrate the potential impacts of storm and water level trends.

2.2. The Generalised Extreme Value model

The Generalised Extreme Value (GEV) distribution has been used extensively for extreme value analysis of hydrological events and specifically for wave heights by Chini et al. (2010), Mendez et al. (2008), Minguez et al. (2010), Guedes Soares and Scotto (2004) and Ruggiero et al. (2010). The GEV encompasses three distributions often referred to as Types I, II and III. The probability density function is given by

$$y = \sigma^{-1} exp\left(-\left(1+k\frac{x-\mu}{\sigma}\right)^{-\frac{1}{k}}\right)\left(1+k\frac{x-\mu}{\sigma}\right)^{-1-\frac{1}{k}} \qquad (1)$$

for $\left(1+k\frac{x-\mu}{\sigma}\right)<0$, where μ is the location parameter, σ is the scale parameter and k is the shape parameter. This traditionally stationary model can be adapted to model non-stationary events by making the GEV parameters time dependent (Katz et al., 2002; Mendez et al., 2008; Minguez et al., 2010). Non-stationarity is usually limited to time varying location and scale parameters $\mu(t)$ and $\sigma(t)$. For example Ruggiero et al. (2010) and Zhang et al. (2004) model the location parameter as a linear function of time and the shape parameter as an exponential function of time. Others who have been interested in cyclic behaviour (such as seasonality) have used trigonometric functions to model the location and shape parameters (Katz et al., 2002; Mendez et al., 2008; Minguez et al., 2010). For the present study we have assumed that the time dependency can be expressed simply as

$$\mu(t) = \mu_0 + \mu_1 t,\ \sigma(t) = \sigma_0,\ k(t) = k_0, \qquad (2)$$

where the location parameter is assumed to be linearly dependent on time and the shape and scale parameters are assumed to be constant based on the findings of Wang et al. (2004). Corbella and Stretch (2011a) identified increasing trends in *Hs*, *T* and the frequency of storm events. However, only the wave height *Hs* was modelled with a

[1] Mean sea level is approximately 1 m above chart datum at this location.

non-stationary GEV model in the present study, while changes in T and D were included through their dependence on Hs as captured by a multivariate statistical model based on copulas and described in Section 5.

2.2.1. Sea level estimating with the GEV model

Sea level can be broadly divided into astronomical forcing and sea level rise. Sea level rise is well documented and the trend is usually described linearly (Mather, 2008 and references therein). A rate of 2.7 mm/year for sea level rise was assumed for the present study based on the work of Mather (2007, 2008). The increase in sea level is included in the GEV through the location parameter and is expressed as

$$\mu_{SLR}(t) = 0.0027t, \tag{3}$$

where $\mu_{SLR}(t)$ denotes the sea level rise component of the location parameter.

The sinusoidal nature of astronomical forcing as well as the numerous cycles makes it more complicated to include in the GEV model. Examples of the long-term astronomical forcings include the 18.6 year nodal cycle due to the regression of the lunar nodes and the 4.5 year cycle when the lunar perigee coincides with the equinox (Pugh, 1987). These two cycles do not have the same phase and the interaction of these two cycles should be included in the model. To reduce the complexity of the model we only consider the 18.6 year nodal cycle in the present study. We include the nodal cycle in the location and scale parameter as described by Mendez et al. (2008), whence

$$\mu_N(t) = \beta_{N_1} cos(2\pi t/18.6) + \beta_{N_2} sin(2\pi t/18.6), \tag{4}$$

$$\sigma_N(t) = \alpha_{N_1} cos(2\pi t/18.6) + \alpha_{N_2} sin(2\pi t/18.6), \tag{5}$$

where $\mu_N(t)$ and $\sigma_N(t)$ denote the nodal cycle component of the location and scale parameters respectively, and β and α describe the amplitudes of the nodal cycle. The nodal cycle components were estimated from simulated tidal data between the years 1980 and 2010.

2.3. Fitting the Generalised Extreme Value model

The fitting of these distributions requires the sampling of extreme events. The simplest sampling method is the annual maximum method which fits a probability distribution to the annual maxima wave heights. The peak over threshold method samples all events exceeding a specific threshold. Unlike the annual maximum method the peak over threshold method allows more than one event per year to be sampled and is therefore commonly used for analysing short data sets. Generally the GEV is used to model block maxima while the generalised Pareto distribution (GP) is used to model data that has a threshold. However, there is no theoretical ground to recommend a specific distribution function for the peak over threshold method (Goda, 2008). For our data set the GEV and the GP provide similar results with the GEV having a superior Akaike information criterion. We therefore use the GEV to model all the parameters of interest. It should be noted that the marginal distributions do not affect the dependence structure modelled by the copulas and may be replaced with any preferred distribution.

The maximum likelihood method of parameter estimation maximizes the joint probability of observing the data in the sample. This intuitive method has been referred to as the most popular and best technique for deriving estimators (Casella and Berger, 1990; Montgomery and Runger, 2003). The maximum likelihood method is popular with statisticians because its characteristics are underpinned by a well developed theory (Goda, 2008). The method was therefore selected for this study.

The significance of non-stationarity in the GEV distributions was evaluated using a log-likelihood ratio test. The test identifies the statistical significance of a trend when compared to a model without a trend. Let M_1 be a model with a trend and M_0 be a model without a trend. If the corresponding log-likelihoods are given by ℓ_{M_0} and ℓ_{M_1} respectively, then the log-likelihood ratio statistic given by

$$LRS = 2\left(\ell_{M_1} - \ell_{M_0}\right), \tag{6}$$

is asymptotically chi-squared distributed with degree-of-freedom equal to the difference between the number of free parameters in the two models. We reject the no trend M_0 hypothesis at a significance level of 95% if LRS exceeds the upper 95^{th} percentile of the chi-squared distribution.

2.4. Event frequency

The non-stationary GEV distribution has been used to model the change in wave height. The annual increase in the frequency of these events still has to be included in the model. The definition of an average recurrence interval for a partial duration series is given by (Goda, 2008; Salvadori, 2004)

$$\tau = \frac{\lambda}{1-p}, \tag{7}$$

where τ is the average recurrence interval, λ is the average event inter-arrival time $1/(D+I)$ (or inverse of the annual average event frequency f_0) and p is the non-exceedance probability of an event. A trend in event frequency can be included by making λ time dependent. Assuming a linear trend we represent the trend in $\lambda(t)$ as

$$\lambda(t) = \frac{1}{f_0 + f_1(t)}. \tag{8}$$

Eq. (7) can then be used to express p as a time dependent function of τ, namely

$$p = 1 - \frac{1}{\tau(f_0 + f_1(t))}. \tag{9}$$

This frequency model can now be used in combination with the non-stationary GEV model to estimate a future wave height for a given recurrence interval based on the trend in frequency and intensity.

2.5. Archimedean copulas

Erosion is not only dependent on wave heights but also on wave period, storm duration, wave angle and sea level. A multivariate statistical model was therefore used to model the dependency between these parameters. Since the physical relationships between the parameters are not expected to change they can be used with both stationary and non-stationary GEV distributions.

The multivariate model was constructed using Archimedean copulas. Copulas are mathematical functions that join or couple multivariate probability distribution functions $F(x_1,\ldots,x_n)$ to their one-dimensional marginal distribution functions $F_1(x_1),\ldots,F_n(x_n)$. For a detailed introduction to copulas refer to Nelsen (2006), Salvadori and De Michele (2010), and De Michele et al. (2007). Using a 2-dimensional case as an example, an Archimedean copula C is defined as

$$C(u,v) = \varphi^{-1}[\varphi(u) + \varphi(v)], \tag{10}$$

where $u = F(x)$ and $v = F(y)$ are marginal distribution functions and φ is the generator function.

Corbella and Stretch (2011b) found that only Hs, T and D of the case study wave data are significantly inter-dependent. Based on this observation they created a fully nested trivariate hierarchical Clayton copula. The same trivariate model was used for the present study. The Clayton copula generator function is given by

$$\varphi(s) = \frac{1}{\theta}\left(s^{\theta} - 1\right) \tag{11}$$

where θ is the dependence parameter and $s \in [0,1]$. The 3-dimensional hierarchical copula has 2 generators, φ_1 and φ_2 and is expressed as

$$C(u_1, u_2, u_3) = \varphi_2^{-1}\left(\varphi_2\left(\varphi_1^{-1}[\varphi_1(u_1) + \varphi_1(u_2)]\right) + \varphi_2(u_3)\right). \tag{12}$$

This model can be used to simulate events conditionally given the expected future wave heights estimated by the non-stationary GEV model. We perform this simulation using the conditional inversion method (De Michele et al., 2007; Nelsen, 2006; Savu and Trede, 2006, 2010). Given the non-exceedance probability of a wave height h the non-exceedance probability of duration d can be estimated from the conditional law G of the bivariate copula as

$$G_2(d|h) = \partial_h C(h, d). \tag{13}$$

The non-exceedance probability of the wave period t can then be estimated conditionally based on the given values of h and d from the bivariate and trivariate copulas as

$$G_3(t|h, d) = \frac{\partial_{h,d} C(h, d, t)}{\partial_{h,d} C(h, d)}. \tag{14}$$

Sampled values for Hs, D and T can then be found by inverting the associated GEV models. It should be noted that Hs, D and T are also dependent on wave direction. Wave direction was not included in the copula model because all the sampled storm events fall between 110° and 180° with an average direction of 147°. There is no significant rank correlation between wave height and wave direction so we assume that all extreme events are equally likely to arrive from any direction between 110° and 180°.

The main advantage of copulas is that they are not limited to dependence described by linear correlation. Dependence measured as a linear correlation is only suitable for a special class of distribution (i.e. elliptical distributions) and its uses outside of these distributions leads to numerous fallacies (see McNeil et al., 2005). We therefore use Kendall's tau rank correlation as a measure of dependence. The Clayton copula interpolates between dependency structures. For the limits $\theta \to 0$ the Clayton copula becomes the independence copula. For $\theta \to \infty$ the comonotonicity copula is produced and for $\theta \to -1$ the Frechet–Hoeffding lower bound is obtained. The Clayton copula therefore interpolates between countermonotonicity, independence and comonotonicity.

2.6. Erosion estimation by numerical modelling

There are numerous numerical models available for estimating cross-shore erosion (Schoonees and Theron, 1995). We limit our analysis to SBEACH (Larson et al., 1990), the Time Convolution model (Kriebel and Dean, 1993), and XBEACH (Roelvink et al., 2009). Although SBEACH has been found to under estimate erosion (Seymour et al., 2005; Zheng and Dean, 1997) it generally provides reasonable predictions (Schoonees and Theron, 1995; Zheng and Dean, 1997). Kriebel and Dean's model is the simplest of the three models and is based on the theory of idealised equilibrium profiles where the water depth

$$h = \begin{cases} -B & x \le -\frac{B}{m} \\ mx & -\frac{B}{m} \le x \le \frac{4A^3}{9m^3} \\ A\left(x - \frac{4A^3}{27m^3}\right)^{\frac{3}{2}} & x \ge \frac{4A^3}{9m^3}, \end{cases} \tag{15}$$

where x is the cross-shore distance offshore, B is the dune height above mean sea level, and A is an empirical coefficient that depends on the sediment settling velocity (Kriebel and Dean, 1993). Storm surge for the Time Convolution model was limited to the sum of the tidal anomaly and wave setup. Assuming saturated wave conditions and ignoring bed shear stresses the maximum wave setup can be estimated as (Callaghan et al. (2008) citing Dean and Dalrymple (1991))

$$\bar{\eta}_{max} = \frac{40 - 3\gamma_b^2}{128}\gamma_b H_b, \tag{16}$$

where $\gamma_b = H_b/h_b$ and H_b is the wave height where wave breaking initially occurs and h_b is the depth that wave breaking occurs. The values of H_b and h_b were estimated for the simulated events in the SWAN model (refer to Section 1).

The Time Convolution model has been previously applied to estimating erosion (e.g. Callaghan et al., 2008). However, it is not intended to accurately reproduce erosion processes but rather to provide a fast and easy method to estimate profile retreat.

The XBEACH model is a relatively new public-domain model that is still under development. Although XBEACH has not yet been tested as extensively as the SBEACH model, it has been used for a number of recent studies and has given satisfactory results (e.g. Hartanto et al., 2011; Roelvink et al., 2009).

The three models were calibrated using the 2007 storm event and verified using the 1998 event. The 1998 event had profile measurements one day before the storm peak and two days after the peak. The 2007 event had measured profiles one month before the event and 9 days after the event. These were the only two storm events that had profile data close enough to the event for calibration purposes.

All the models predict erosion of the profile but the sediment remains within the model domain and there is no net loss of sediment. Therefore, only the erosion above mean sea level (MSL) was calculated and compared with the field data.

The wave events simulated by the statistical model are based on recordings from the waverider buoy and so we use the numerical wave model SWAN (Simulating WAves Nearshore) to transform these events to the required locations. The significant wave height and storm duration estimated from the statistical model are used to create an idealised time series of a storm (Fig. 7). This time series is transformed to a nearshore time series by the SWAN model and the transformed time series is used in the cross-shore models.

Since the statistical model did not include wave direction we simplify the erosion simulations by setting all the offshore deepwater wave directions equal to 145°.

2.6.1. The SWAN model

The SWAN model used in this study was set up using three grid resolutions. The largest, offshore rectangular grid had approximately 1000 m × 1000 m grid cells. A medium grid had 400 m × 400 m cells and the smallest curvilinear grid had approximately 100 m × 50 m cells. The statistically simulated wave conditions were transformed to 20 m water depths in front of each of the selected profile locations. Hind-cast data for the two storms were obtained from the WAVEWATCH-III global wave model (Tolman, 1999) and used as offshore boundary conditions to calibrate and verify the SWAN model with the wave recording buoy data.

2.7. Summary

We have outlined the creation of a time dependent statistical model and have described four numerical models. We now summarise the section by providing an algorithm for the process used to quantify future erosion.

1. Sample storm parameters *Hs*, *D*, *T* and λ using the POT method and the storm definition given in Section 1.
2. Identify appropriate functions to model any trends in *Hs* and λ.
3. Fit a non-stationary GEV distribution to *Hs*.
4. Fit GEV distributions to *D* and *T* using the maximum likelihood method.
5. Create a copula model of *Hs*, *D* and *T*.
6. Estimate the time dependent non-exceedance probability p (Eq. (9)) of *Hs* corresponding to a 31-year recurrence interval τ (the 2007 event) for a forecast of 10, 25, 50 and 100 years.
7. Simulate 1 000 000 non-exceedance probability samples, at specified non-exceedance probabilities of h, for storm duration d and wave period t from the conditional laws of the bivariate (Eq.(13)) and trivariate copulas (Eq. (14)).
8. Calculate an event equivalent to the probability levels of the 2007 event from the 1 000 000 samples of d and t.
9. Estimate the value of *Hs*, *D* and *T* from the inverse of the cumulative GEV distribution with the time dependent location parameter (Eq. (2)).
10. Simulate sea levels for a 10, 25, 50 and 100 year forecast from the inverse of the cumulative GEV distribution using the time dependent parameters (Eqs. (3) to (5)).
11. Use the simulated values of *Hs* and *D* to create an idealised storm time series with the maximum significant wave height occurring at half the storm duration (Fig. 7).
12. Use the simulated sea level and a sine function to create a time series of the tide. Make the high tide coincide with the maximum wave height.
13. Calibrate the SWAN, SBEACH, XBEACH and Time Convolution model using past storm events.
14. Use the SWAN model to transform the simulated wave events to the required locations and to calculate the wave breaking heights and water depths for the Time Convolution model.
15. Use the SBEACH, XBEACH and Time Convolution model to estimate the erosion corresponding to the simulated wave events.

The shoreline response models require *Hs*, *T*, *D*, wave direction and water level as inputs. Wave direction could not be included in the statistical model because the 7 years of data is too short to establish a trend or dependence between the other parameters. We therefore use the most common wave direction of south east as a constant wave angle in the models.

3. Results

3.1. Trend analysis

As previously mentioned we limit our analysis of trends to *Hs* and water levels. We then use the dependency between the storm parameters *Hs*, *D* and *T* to estimate the associated changes of *D* and *T*. Water levels were modelled separately because Corbella & Stretch (2011b) found them to be independent of the other storm parameters.

3.1.1. Trends in significant wave height

The observed trend in the significant wave height was not found to be statistically significant at a 95% confidence level (Corbella and Stretch, 2011a). The difficulty in establishing trends from a relatively short data set is illustrated by the return period estimates using the stationary GEV model in Fig. 3. The limited data in the upper tail causes the bootstrapped 95% confidence interval to become large very quickly. Despite the uncertainty, the observed trend in *Hs* was modelled and is shown in Fig. 4. The rate of increase of significant wave heights exceeding 3.5 m was estimated to be 0.02 m/year. This trend was incorporated into the GEV model using the location parameter as described in Section 2. The negative log-likelihood method estimated the rate of change of the location parameter to be 0.0057 m/year. The log-likelihood ratio test between the stationary and non-stationary models confirmed that the trend in *Hs* was not statistically significant. The 0.02 m/year trend is similar to the trends identified by Theron et al. (2010), Ruggiero et al. (2010), Dodet et al. (2010), Bacon and Carter (1991), but because there is limited statistical confidence in the data we analyse both the 0.0057 m/year and 0.020 m/year trends.

The significance of an increasing wave height is illustrated in Fig. 5. The plot shows how the same recurrence interval corresponds to larger events as time proceeds.

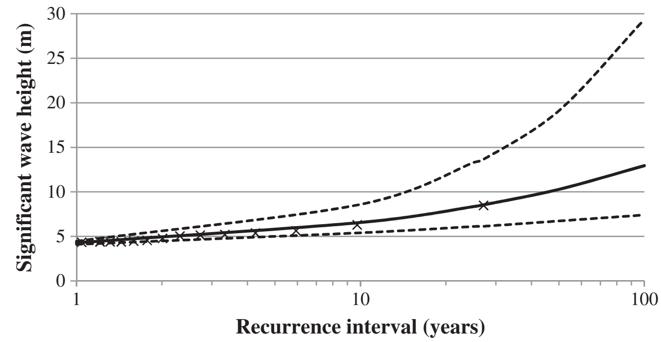

Fig. 3. The recurrence intervals of significant wave height and the bootstrapped 95% confidence. The crosses show the empirical recurrence intervals.

3.1.2. Trends in storm frequency

The trend in storm frequency was estimated from the occurrence of wave heights exceeding 4 m. It was found that the average of $f_0 = 3$ events/yr was increasing at an average rate of $f_1 = 0.01$ events/yr.

3.1.3. Trends in water level

Unlike the GEV model of *Hs* which used the POT method, the GEV for the water levels was fitted to the annual maxima tide levels. As previously mentioned an estimate of 0.0027 m/year was assumed for sea level rise. Similar to Mendez et al. (2007) the contribution of the scale parameter to the nodal cycle was found to be negligible. The amplitude of the nodal cycle $(\beta_{N_1}{}^2 + \beta_{N_2}{}^2)^{1/2} = 0.03$ is similar to Mendez et al. (2007) and Sobey (2001). Note that $\beta_{N_1} = 0.004$ and $\beta_{N_2} = 0.03$ while the nodal

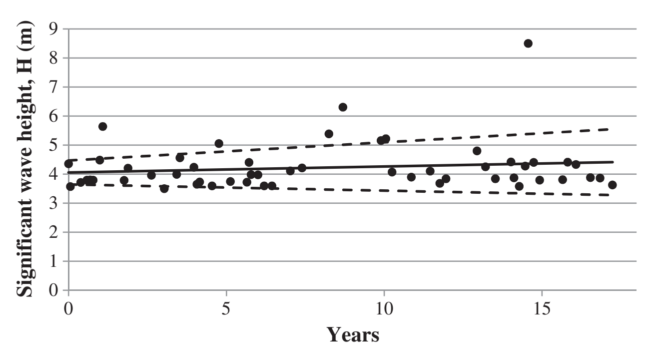

Fig. 4. The significant wave height events exceeding 3.5 m between 1992 and 2010 and their linear regression shown by the solid line and the 95% confidence intervals shown by the dashed lines.

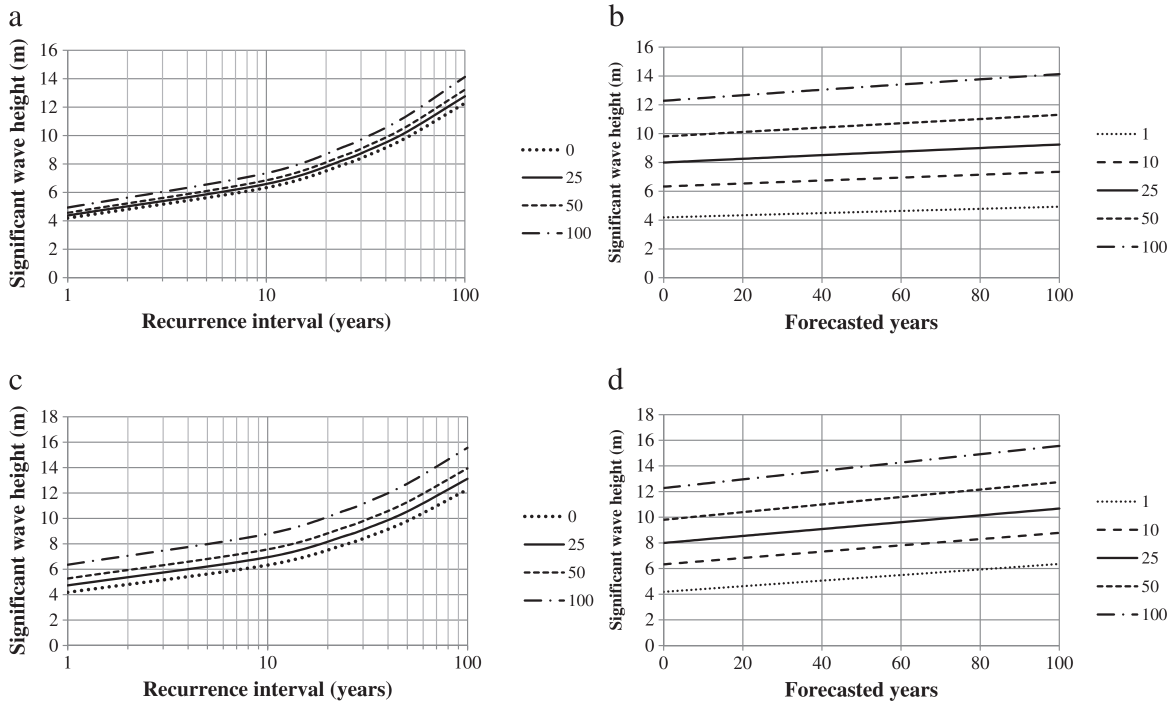

Fig. 5. The 100 year forecasted increase in significant wave heights for recurrence intervals between 1 and 100 years for: (a and b) a 0.0057 m/year increase in *Hs* and (c and d) a 0.02 m/year increase in *Hs*.

cycle contribution in the scale parameter ($\alpha N_1, \alpha N_1$) is not significant. Fig. 6 shows the linear trend in sea level, the sinusoidal trend in the nodal cycle, and the combination of the two trends in the GEV model.

3.2. Simulated events

Table 1 shows the simulated events. The copula model incorporates the dependence between the significant wave height, duration and peak period. This dependence allows the trends in duration and peak period to be modelled conditionally on the trend in significant wave height. Table 1 shows an increasing trend in duration. The simulated rates of *D* and *T* are not consistent with the findings of Corbella and Stretch (2011a). The reason for this is because the trivariate copula has no upper tail dependence and does not represent the average trend of the data set. If an event in the centre of the distribution, such as a wave height of 5 m was used, then there would be a larger change in *D* and *T*. For a detailed explanation of this concept the reader is referred to Corbella and Stretch (2011b). This concept of tail dependence makes the use of copulas very powerful when forecasting conditionally dependent events.

Fig. 7 illustrates the evolution of an idealised 2007 storm event over time. If we define the storm magnitude as the area under the plots in Fig. 7 it can be seen that the overall storm magnitude increases with time. This increase in combination with the increase in peak period means that the total wave power will increase. Furthermore this increased storm magnitude and wave power are able to act further inland as a result of the increase in water level, which will in turn increase erosion.

3.3. Comparison of the numerical erosion models

We do not present the Time Convolution results as a goodness of fit because the method only shifts an equilibrium profile that has been fitted to a historic profile. Instead the results are presented in Table 2 as relative percentage errors between the measured and modelled retreat and erosion volumes.

The average error for the two events and for all the profiles is 48%. This may be an acceptable initial estimate but is not a suitable alternative to a process-based model. The profiles that show the best results are those that have the closest approximation to the equilibrium profile described by Eq. (15). The limitation of assuming a constant wave breaking height is one of the reasons why the erosion of profiles F and 13 is over estimated in Table 2.

SBEACH is a more sophisticated model than the Convolution model but because its calibration parameters are limited it does not have the same flexibility as XBEACH. Fig. 8 shows the modelled beach response of profile A for both the SBEACH and XBEACH models. XBEACH modelled the response of profile 13 worse than any other profile while it was the best of the SBEACH simulations. SBEACH estimates the correct profile shape but shows no net erosion while XBEACH has a similar shape and over estimates the erosion by almost 20%. The SBEACH results are not tabulated as they mostly show no net erosion above the 0 m MSL contour.

The XBEACH calibration results are shown in Table 3. The model was calibrated on the 2007 event and predicts the erosion volumes within 10% on average. The calibration was verified with the 1998 event and simulated erosion volumes were between 1% and 57% of the measurements with an average error of 30%. XBEACH is the preferred of the three models and will be used to estimate the erosion of the forecasted events. The calibration is acceptable given that it does not include longshore currents and that we are essentially interested in relative erosion volumes.

3.4. Predicted erosion

The forecast storm conditions and modelled erosion trends are shown as relative percentage changes in Fig. 9. The significant wave height and water levels were modelled with increasing trends with the 18.6 year nodal cycle included in the water level trend. The peak period and duration have a minor increasing trend, neither of which contributes to the estimated increase in erosion. The average annual erosion trend of the profiles was estimated to be 0.14%/year/storm and 0.20%/year/storm as a result of the 0.16% and 0.32% increase per year in wave height respectively and the 0.21%/year increase in water level.

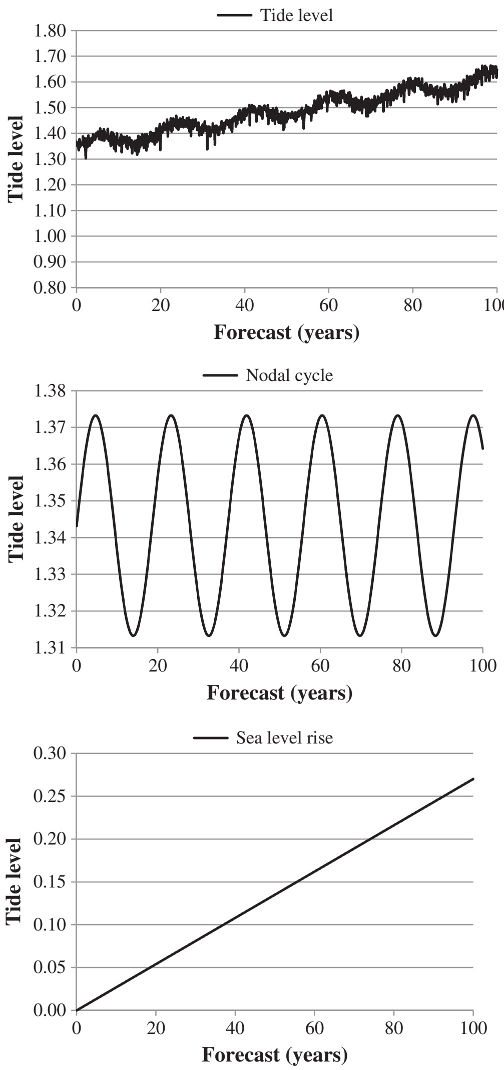

Fig. 6. A 100-year forecast of sea levels from the non-stationary Generalised Extreme Value model showing the nodal cycle and sea level rise components of the trend.

4. Model limitations

The non-stationary GEV and copula method proposed here for forecasting storm events is more appropriate than simulating events based purely on marginal distributions because it includes the dependence between parameters. The method however assumes that the dependence relationships that exist between storm parameters do not change over time. This may not be correct because there could be changes in the meteorological forcing processes. For example Durban's storm waves are produced by either cut-off lows, cold fronts or tropical cyclones. It is estimated that only 5% of Durban's storm waves are produced by cyclones and so the current model is mainly representative of a mixture of cold fronts and cut-off lows. Although the relationship between the meteorological forcing and storm durations is still unclear at the case study location, it appears that the larger wave heights and longer period waves are associated with cut-off lows. If there is a trend in only a single forcing process it may eventually affect the relationship modelled by the multivariate copula. If only one of the forcing processes is increasing our assumption of a constant dependence relationship will be incorrect.

Table 1
The storm parameters of the 2007 event significant (wave height *Hs*, duration *D*, peak period *T* and water level *WL*) and the equivalent event storm parameters forecasted 10, 25, 50 and 100 years. Values for Hs trends of both 0.0057 m/yr and 0.02 m/yr (bracketed) are given in the table.

Forecast (years)	0	10	25	50	100
Hs (m)	8.5	8.64 (8.78)	8.84 (9.20)	9.18 (9.89)	9.82 (11.3)
D (hrs)	55.4	55.5 (55.6)	55.6 (55.9)	55.7 (56.1)	55.9 (56.4)
T (s)	16.6	16.6 (16.6)	16.6 (16.6)	16.6 (16.6)	16.6 (16.6)
WL (m)	1.33	1.36	1.42	1.45	1.62

The proposed method of conditionally simulating peak wave period from a significant wave height should be applied cautiously. In the case study presented here the method performs well because of the forcing processes described above. If local storm conditions produce extreme waves the proposed method will overestimate the peak wave period and thus the extreme storm conditions. If there is a possibility of this occurring at the location of interest the copula based simulation of peak wave period may be replaced by the method described by Monbet and Prevosto (2001) and references cited therein.

Further caution must be observed in the use of Archimedean Copulas. Copulas provide a very general model of dependence and although they have found various successful applications their generality makes them difficult to estimate from data. Copulas that are derived from multivariate distributions (e.g. the Gaussian copula) provide powerful models because the marginal distributions and the dependence structure can be disentangled and handled independently. Archimedean copulas however do not stem from multivariate distributions and are designed in part for mathematical tractability. Therefore their appropriateness as natural models for dependence should be verified for each application.

The erosion simulations have been performed from a constant antecedent profile. This method does not allow for long term retreat due to sea level rise. The method also neglects any trends in wave direction. The long term erosion effects of changing wave directions are an important factor (Zacharioudaki and Reeve, 2011) and a notable weakness in the current study. Although the method is not fully realistic it allows the effects of storms to be quantified separately to that of long term sea level rise and wave direction effects. Since cross-shore erosion processes are dominant during storms the absence of wave direction trends does not significantly affect the quantification of storm erosion trends.

The trends used for the case study are based on analysis of limited data and are not statistically significant. This is expected to be a common problem since most areas in the world have short wave data sets. The absence of statistically significant trends is not a sufficient justification to dismiss the inclusion of such trends in medium to long term planning. The methods outlined in this paper are intended to allow potential future impacts to be quantified so that they can be assessed as part of a holistic planning and design process.

5. Discussion

Three cross-shore erosion models have been evaluated for predicting the impacts of wave and water level trends. The Time Convolution model (Kriebel and Dean, 1993) was the simplest model evaluated and showed an average relative error between the measured and modelled erosion volumes of 48%. SBEACH is considered a far more sophisticated model than the Time Convolution model. SBEACH modelled the erosion volumes well with regards to shape but the erosion volume was largely balanced by the accretion volume above 0 m MSL and therefore yielded no net erosion. The Time Convolution model provides reasonable initial estimates of profile erosion relative to its simplicity. SBEACH is an appealing model because it has an easy to use graphical user interface and the simulation times are short. The user interface unfortunately removes a degree of flexibility. The calibration parameters are also limited to the Transport Rate Coefficient and the Coefficient for Slope-Dependent term (Sommerfeld et al., 1996). XBEACH gave the best calibration results and was therefore selected for predicting future trends. A disadvantage of XBEACH is that the simulation times are significantly longer than the other two models. Simulation times can be reduced to some extent by using a morphological time factor. The required length of the XBEACH

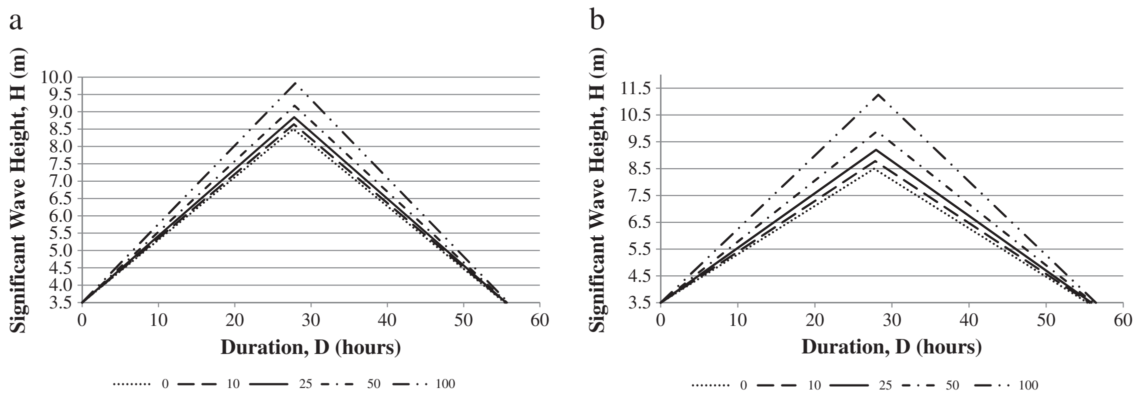

Fig. 7. Plots of the idealised 2007 equivalent storm event over 10, 25, 50 and 100 year forecast for (a) a 0.0057 m/year increase in *Hs* and (b) a 0.02 m/year increase in *Hs*.

simulations necessitates the use of a multivariate statistical model. The more pragmatic engineering approach (e.g. Reeve, 1998) would require a lengthy time series as an input into XBEACH. Although the results of the engineering approach can be simply analysed as univariate erosion the extensive XBEACH simulations make the engineering approach impractical.

Average erosion was estimated to increase at a rate of 0.14%/year/storm and 0.20%/year/storm for increases in wave height *Hs* of 0.0057 m/yr and 0.02 m/yr respectively. Corbella and Stretch (2011a) found that most of the Durban beach profiles have a long term erosion trend and they identified numerous reasons for the erosion trends. Although this paper has only analysed the erosion associated with storm trends at a single probability level, the results provide an indication of general storm erosion trends. We can therefore estimate the proportion of long-term erosion due to either trends in storm characteristics or to sea level rise. Table 4 shows the long term erosion trends estimated from historical profile data compared to the erosion estimated due to storm and water level trends. Table 4 shows that the storm and water level trends potentially contribute 25% to 42% of the overall erosion trend. The remaining 58% to 75% of the erosion can be attributed to a decrease in sediment budget and long term sea level rise. Corbella and Stretch (2011a) suggested that the decrease in sediment budget is due to a combination of sediment mining and trapping by dams and the episodic nature of large flood events. These results should be interpreted cautiously. The simulations show that the maximum potential contribution to long term erosion may be the result of storm and sea level trends. However, these trends may not contribute to long term erosion at all if there is sufficient recovery time between the storms for the beaches to recover to their pre-storm volumes. Corbella and Stretch (2012) found that on average the shoreline recovery takes 2 years, regardless of the storm magnitude. In this regard, based on our current model and estimates, it is not anticipated that storm trends will contribute to long term erosion for the next 100 years. Sea level rise on the other hand will influence long term erosion based on the Bruun Rule. Corbella and Stretch (2011a) found that the Bruun Rule attributes 75% of the current beach erosion to sea level rise. The combination of sea level rise and storm trends is therefore likely to become an issue prior to the year 2100, without storm trends contributing to long term erosion.

Table 2
Relative errors of the Convolution method for profile retreat and volume erosion fro profiles A, C, D, F and 13.

Storm event	Profile	Retreat error (%)	Volume error (%)	Average error (%)
1998	A	−66	−6	36
	C	9	−35	22
	D	−61	24	42
	F	101	−49	75
	13	−58	95	76
2007	A	−67	−38	52
	C	75	−47	61
	D	−57	−44	50
	F	−35	−29	32
	13	−53	−19	36

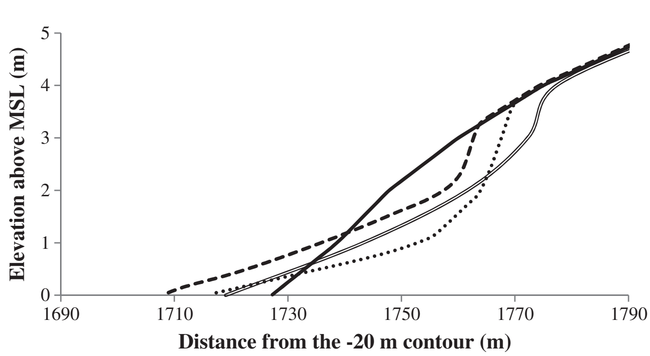

Fig. 8. A model comparison of the response of profile 13 to the 2007 storm event. The solid line shows the pre-storm profile and the double line shows the post-storm profile. The dashed line is the SBEACH simulated storm response and the dotted line is the XBEACH simulated storm response.

6. Conclusion

In this paper we have introduced an integrated modelling approach for assessing future coastal erosion trends under changing climatic conditions. The method combines a multivariate, copula based, non-stationary statistical model for storm waves with deterministic shoreline response models.

The non-stationary GEV model is a useful means for forecasting time dependent wave parameters. Coupling non-stationary GEV models via copulas allows the time dependent parameters to be modelled

Table 3
XBEACH 1D model relative profile erosion volume errors and Chi-squared statistics for profiles A, C, D, F and 13.

Storm event	Profile	Volume error (%)	Chi-squared (χ^2)
1998	A	−31	0.98
	C	36	3.94
	D	57	0.63
	F	−27	4.93
	13	1	0.86
2007	A	−9	2.17
	C	4	1.00
	D	−4	0.64
	F	−7	0.72
	13	19	0.45

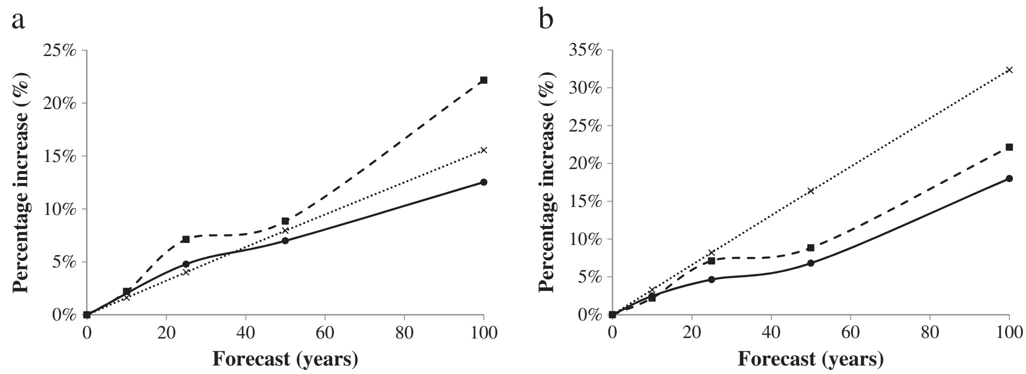

Fig. 9. The forecast percentage increase of: significant wave height shown by the dotted line; water level shown by the dashed line and erosion shown by the solid line.

Table 4
Comparison between long-term profile erosion and erosion estimated from storm and water level trends as percentage relative change. The percentage contribution of the estimated erosion to the long-term erosion is also shown.

Profile	Long-term annual erosion (%)	Storm/water level erosion (%)		Contributions to erosion (%)	
		0.0057 m/year	0.02 m/year	0.0057 m/year	0.02 m/year
A	NA	0.12	0.17	NA	NA
C	0.32	0.10	0.20	31	61
D	−0.21	0.13	0.13	NA	NA
F	1.43	0.19	0.22	14	16
13	0.57	0.18	0.28	32	50
Averages	0.53	0.14	0.20	25	42

conditionally based on the dependence between parameters. This paper has used both of these methods to create a multivariate statistical model of a time dependent sea state.

The statistical model has been used to estimate future storm erosion trends for a South African case study. The investigation of three morphological models (Time Convolution, SBEACH and XBEACH) showed the XBEACH model to have the best results while the Convolution model provided a simple means to find reasonable erosion estimates. Callaghan et al. (2008) concluded that their reliance on the Kriebel and Dean (1993) model was a limitation to their full temporal simulation method. The use of the XBEACH process-based model is an improvement on the Convolution model. We have also improved the statistical description of storm events by including the dependence between wave height, wave period and storm duration using copulas. The predicted future erosion due to storm and sea level trends was estimated to increase at a rate of 0.14%/year/storm and 0.20%/year/storm as a result of the 0.0057 m/yr and 0.02 m/yr increase wave height respectively. It has been estimated that storm trends are unlikely to contribute to long term erosion prior to the year 2100 while it is plausible that sea level rise is already contributing to long term erosion.

The methods presented in this paper should be useful for medium to long term planning by coastal managers and decision makers.

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