[entry]nyt/global/

1 Bélanger surface profile with zero bed slope

Quality Assurance

Date	Author	Initials	Review	Initials	Approval	Initials
08 Dec 2017	Andries Paarlberg		Arthur van Dam		Aukje Spruyt	
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Version information

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Purpose

The purpose of this validation case is to examine the performance of D-Flow FM for a schematized channel flow simulation. For stationary flow through a river with a rectangular cross-section, the Bélanger surface profile equation can be utilized to compare the numerical solution with.
In the D-Flow FM test bench, a case is available where a numerical approximation of the surface profile based on the Bélanger equation is implemented. For use in further test cases we test a Matlab-version of this implementation for bed slope $i_b \geq 0$ The case for flat bed slope ($i_b = 0$) is of specific interest, since for a number of test cases, it is convenient to use a flat bed (without a bed slope).

Linked claims

Claims that are related to the current test case are:

- Water levels in 1D-model are identical to Quasi2D-model for flat bed.
- Water levels are identical to semi-analytical solution.
- Matlab implementation of Bélanger approximation is equal to that in D-Flow FM.

Approach

Base case: `/DSCTestbench/cases/e02_dflowfm/f03_advection/c010_belanger/`
This case is set-up for arbitrary bed slopes ($i_b \geq 0$). Here we investigate how the Matlab code performs w.r.t. the D-Flow FM code for the case with a horizontal flat bed ($i_b = 0$) and a bed slope of $i_b = 4\text{x}10^{-4}$ m/m.

A straight channel with a rectangular cross-section is defined. Given an inflow discharge Q, a channel width B, a bottom slope i_b and a Chézy friction factor C, the distance between the free surface profile and the bed profile can be described by the Bélanger equation for d as the water depth:

$$\frac{\mathrm{d}d}{\mathrm{d}x} = i_b \frac{d^3 - d_e^3}{d^3 - d_g^3} \tag{1}$$

with d_e the equilibrium depth and d_g the limit depth (associated with $\mathrm{Fr} = 1$) following:

$$d_e = \left(\frac{Q^2}{B^2 g}\right)^{1/3} \quad \text{and} \quad d_g = \left(\frac{Q^2}{B^2 C^2 i_b}\right)^{1/3}. \tag{2}$$

Given a certain inflow discharge Q_{in} and a certain outflow water level condition h_{out}, the surface profile can hence numerically be estimated in the most simple way as:

$$\frac{d_i - d_{i-1}}{\Delta x} = i_b \frac{d_i^3 - d_e^3}{d_i^3 - d_g^3}, \tag{3}$$

having $d_i = h_{out} + i_b L$ at the outflow boundary. This, in fact *semi-analytical*, solution can be used for comparison.

For zero bed slope we use a slightly different formula for the estimation of the surface profile, which is independent of the equilibrium depth:

$$\frac{d_i - d_{i-1}}{\Delta x} = -\frac{c_f d_g^3}{d_i^3 - d_g^3}, \tag{4}$$

where:

$$c_f = \frac{g}{C^2} \tag{5}$$

Model description

Relevant files for the case with zero bed slope are:

- MDU-file: belangerflat1d2d_rst.mdu
- Grid-file: 5001d2dflat_net.nc
- External forcings file: flat.ext

For the case of non-zero bed slope, the bed levels are set *inside* D-Flow FM (only implemented for 2D) by using a 2D-grid file with NaN-values and specifying the bed slope directly in the

- MDU-file: belanger_AP_rst.mdu
- Grid-file: 5002dmis_net.nc

The 1D and 2D channels have equal length, see figure below.

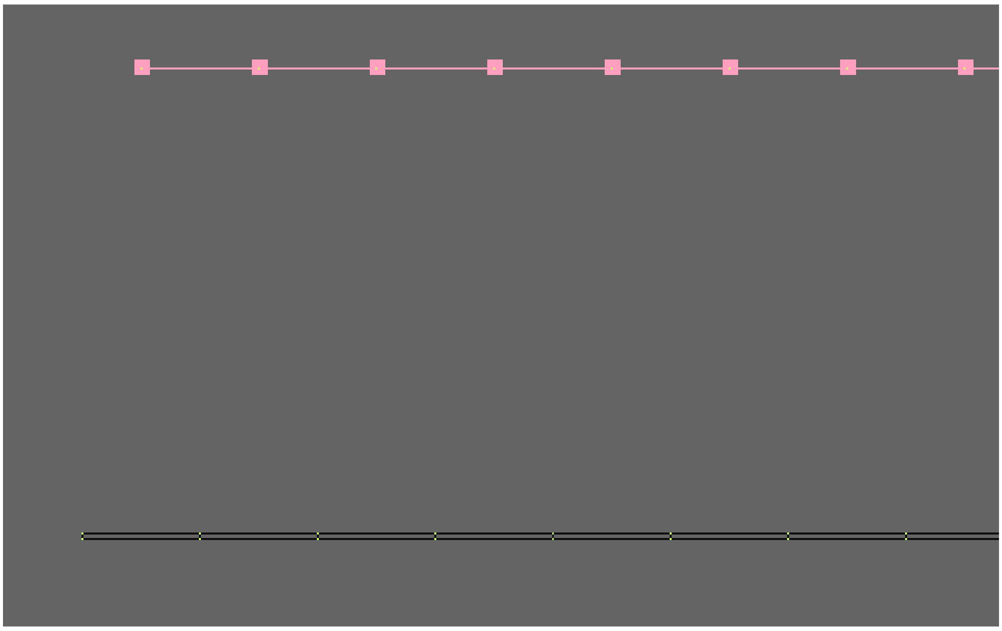

Figure 1: Figure of the layout of the model

The 2D computational domain has the sizes $L \times B = 100$ km $\times$ 20 m. The grid consists of 200×1 cells. The cell size is 500×20 m^2 everywhere. The bed slope i_b is 0. The inflow discharge is $Q = 600$ m^3/s. The Chézy coefficient is $C = 60$ m$^{1/2}$/s. The outflow water *level* is set equal to -0.12600 m (w.r.t. reference); the water *depth* at the downstream boundary is approximately 10 meters (differs per bed slope). Recall that the water depth is computed as the difference between the upstream water level (computed at the *cell center*) and the bed level at the velocity point (computed at the *cell face*), invoking a $\Delta x/2$ spatial shift. In the computational model, the bed level at the outflow boundary is equal to -10 m+NAP. Therefore, the specified water level holds at a distance $\Delta x/2$ outside the grid (mirrored location).

The case is run for 1 day, starting from a restart-file, to ensure a numerically converged solution.

Results

In D-Flow FM, the semi-analytical solution is implemented. When running the model in the interactive GUI, the deviations from the semi-analytical solution are shown on the screen, which are, for the case of a zero bed slope, approximately:

- 2D @upstream: 0.0157 m = 1.57 cm
- 1D @upstream: 0.0220 m = 2.22 cm
- 2D @ 80 km: 0.0224 m = 2.24 cm
- 1D @ 80 km: 0.0296 m = 2.96 cm

For the case of non-zero bed slope, the difference are sligthly larger, with a maximum of approximately 5 cm at the upstream boundary. In all cases, D-Flow FM is slightly lower than the analytical solution.

The result from D-Flow FM for the water *depth* is shown in the figures below in combination with its semi-analytical equivalent (here found with a Matlab code). The semi-analytical solution is based on the equation for the Bélanger surface profile.
Note!: to correctly reproduce the semi-analytical solution, Δx should be chosen small enough (here 1 m).

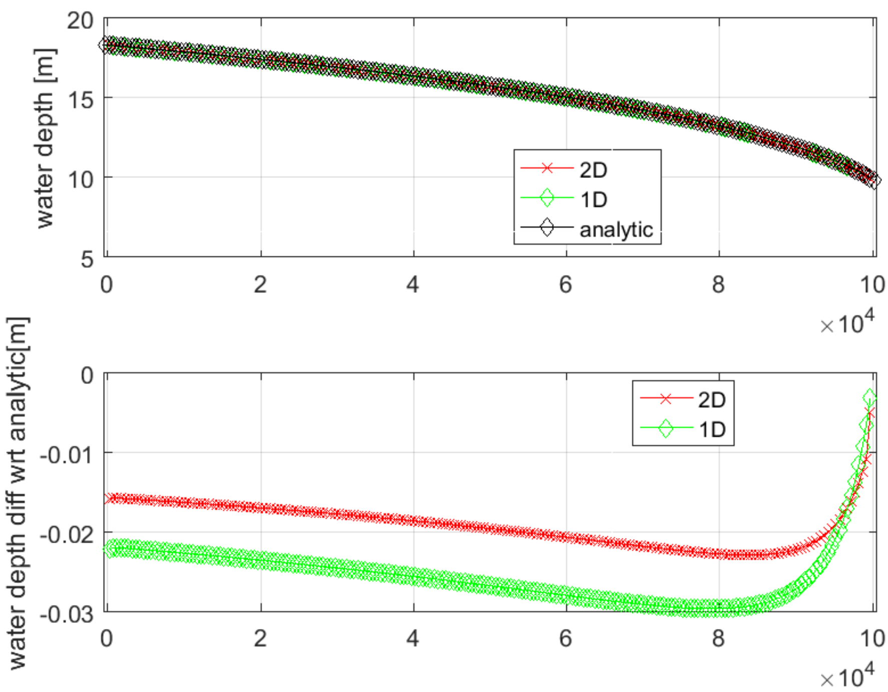

Figure 2: Comparison of the numerical solution and the semi-analytical solution for the water depth for zero bed slope ($i_b = 0$).

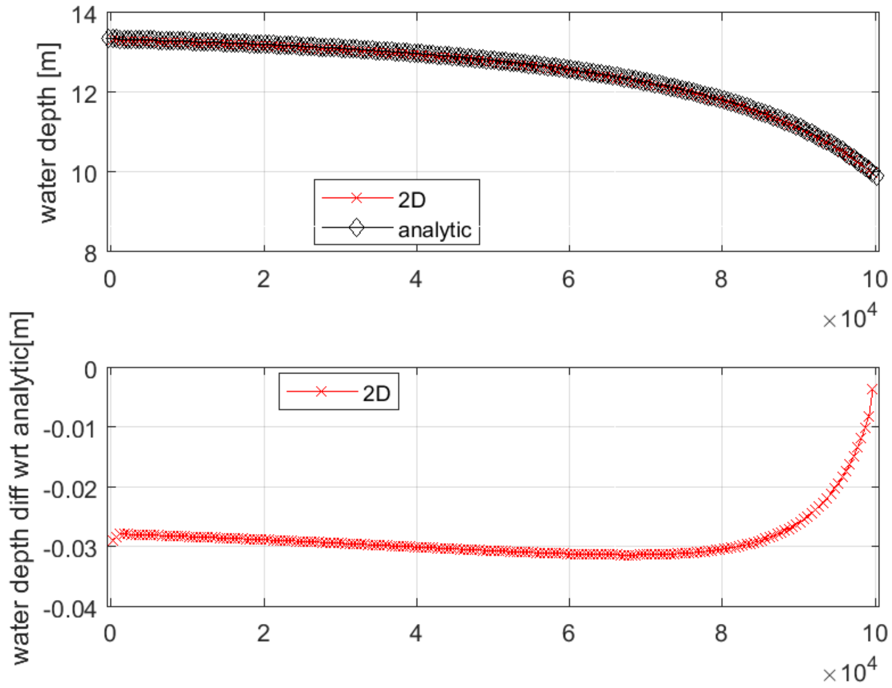

Figure 3: Comparison of the numerical solution and the semi-analytical solution for the water depth for $i_b = 4\text{x}10^{-4}$ m/m.

The difference in water depth compared to the analytical solution are equal to the difference computed in D-Flow FM. From validation document: root-mean-square difference between the numerical outcome from D-Flow FM and the semi-analytical solution is in the order of 10^{-3} m (for a channel of 100 km).

Conclusion

Water levels in the 1D-model are identical to those in the Quasi2D-model for the case of zero bed slope. For non-zero bed slope ($i_b = 4\text{x}10^{-4}$) the surface profiles found with D-Flow FM and Matlab are nearly identical.
This gives confidence to use the Matlab-code for $i_b \geq 0$ for further test cases.